Asymptotic Analysis

Big-Oh: Basic Examples

Design and Analysis of Algorithms I
Example #1

Claim: if \( T(n) = a_k n^k + \ldots + a_1 n + a_0 \) then

\[ T(n) = O(n^k) \]

Proof: Choose \( n_0 = 1 \) and \( c = |a_k| + |a_{k-1}| + \ldots + |a_1| + |a_0| \)

Need to show that \( \forall n \geq 1, T(n) \leq c \cdot n^k \)

We have, for every \( n \geq 1 \),

\[ T(n) \leq |a_k| n^k + \ldots + |a_1| n + |a_0| \]
\[ \leq |a_k| n^k + \ldots + |a_1| n^k + |a_0| n^k \]
\[ = c \cdot n^k \]
Example #2

Claim: for every \( k \geq 1 \), \( n^k \) is not \( O(n^{k-1}) \)

Proof: by contradiction. Suppose \( n^k = O(n^{k-1}) \)
Then there exist constants \( c, n_0 \) such that
\[
n^k \leq c \cdot n^{k-1} \quad \forall n \geq n_0
\]
But then [cancelling \( n^{k-1} \) from both sides]:
\[
n \leq c \quad \forall n \geq n_0
\]
Which is clearly False [contradiction].