Introduction to Algorithms

• Slides by Avi Wigderson

Evaluating how good (how efficient) an algorithm is How does the number of basic steps of an algorithm increase with the data size (input length) ? input



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|---|---|---|----|---|---|---|---|---|----|----|----|----|----|---------|----|----|----|----|
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| | | | | | | | | | | 12 | | | 10 | | | 1 | | 13 |
| | | | | | | | | | 3 | _ | _ | 15 | 2 | | | 14 | | _ |
| | | 8 | 6 | | | | | | 13 | | - | | 8 | _ | _ | 10 | | 12 |
| | | | | | | | 6 | | - | 11 | ' | 6 | | | 40 | 16 | | |
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| 2 | | | | 4 | | 7 | | | | | 12 | 4 | | 6 | 16 | | 13 | |
| 3 | 6 | | | 2 | 9 | | | | | 3 | | | 12 | | | | 6 | |
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| 13 | | | | 8 | | | 10 | | 12 | 2 | | 1 | 15 | | |
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| | 13 | | | 15 | | 3 | | | 14 | 8 | | 16 | | | |
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| | | 12 | 4 | | 6 | 16 | | 13 | | | 7 | | | | 5 |
| | 3 | | | 12 | | | | 6 | | | 4 | 11 | | | 16 |
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| 11 | 1 | 15 | 9 | | | 13 | | | 2 | | | | 14 | | |
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| Corr | Complexity | | | | | | | | | |
|----------|--------------------|------|--|--|---|---|---|---|---|--|
| | | | | | | 1 | 1 | 1 | | |
| | | | | | 1 | 2 | 3 | 4 | 5 | |
| # digits | # s [.] | teps | | | | 6 | 7 | 8 | 9 | |
| | Hindu | | | | 1 | 9 | 1 | 3 | 4 | |
| 1 | 6.1 | | | | | | | | | |
| 5 | <u>6</u> ∙5 | | | | | | | | | |
| 10 | <mark>6</mark> ∙10 | | | | | | | | | |
| 100 | 6.100 | | | | | | | | | |
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| N | 6·N | | | | | | | | | |
| Ν | ~N | | | | | | | | | |

| Com algo | paring prithm |) IS | | | | | | | |
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| N | ~N | ~10 ^N | | | | | | | |
| | | | | | | Ad Gro Whi Y: X: end | ditio eek a ile Y>(=Y-1 :=X+1 While | n: Ilgori D | ithm |



| Com algo | paring rithm |) IS | | | | | | | | |
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| # digits | # s | teps | | | 1 | 2 | 3 | 4 | 5 | |
| | Hindu Greek | | | | | 6 | 7 | 8 | 9 | |
| N | ~N | ~10 ^N | | | | | | | | |
| Hindu: optimal – "It was the best of times" Graek: terrible –"it was the worst of times" | | | | | | | | | | |
| Complexity of a function = Complexity of its best algorithm | | | | | | | | | | |



Complexity of functions

| Providence bandla |
|---|
| [gradesenool] 1) [schoenhage-strassen] |
| r algorithm? Grade-school multiply algorithm |
| X ******** |
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