Problems

1. (10 pts) (this problem was suggested by Paul Beame) Modify Karatsuba’s algorithm based on splitting the polynomials into 3 pieces instead of two. Base your algorithm on a method for multiplying two degree 2 polynomials that uses 6 multiplications. What is the running time of this algorithm? How does the running time of your new algorithm compare to that of Karatsuba’s algorithm?

2. (10 pts) Describe an $O(n)$-time algorithm that, given a array $A$ of $n$ distinct numbers and a positive integer $k \leq n$, determines the $k$ numbers that are the closest to the median of $A$. For example, if $A$ is $\{10, 2, 3, 7, 9\}$, the median in this case is 7 and if $k = 2$, the output is the list $\{9, 10\}$. Note that if two numbers are at the same distance from the median, and we can’t choose both, either one is fine.

3. (10 pts) Chapter 5, Problem 5.

Extra Credit

1. (10 pts) Chapter 3, Problem 6. Show how to multiply two degree 2 polynomials using only 5 multiplications. If you use this in a modified version of Karatsuba’s algorithm what running time would you get?