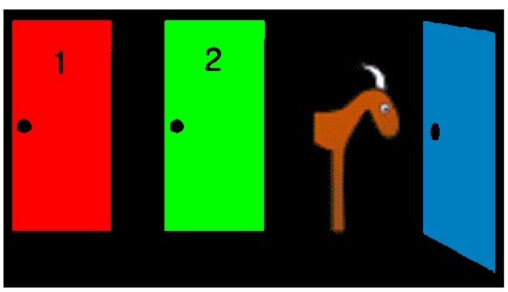
Uncertainty in AI: The Joint Distribution

CSE 415: Introduction to Artificial Intelligence
University of Washington
Spring, 2017

Outline

- The Monty Hall Problem revisited
- Joint probability distributions
- Marginal distributions
- Factored joint probability distributions
- Bayes nets
- Benefits of Bayes nets for expert systems

The Monty Hall Problem (revisited)



Joint Probability Distribution for the Monty Hall Problem

Prize in	You choose	Host opens	P	Payoff if no switch	Payoff if switch
R	R	G	1/18	1	0
R	R	B	1/18	1	0
R	G	B	1/9	0	1
R	B	G	1/9	0	1
G	R	B	1/9	0	1
G	G	R	1/18	1	0
G	G	B	1/18	1	0
G	B	R	1/9	0	1
B	R	G	1/9	0	1
B	G	R	1/9	0	1
B	B	G	1/18	1	0
B	B	R	1/18	1	0

Discussion

Marginal probability of winning, never switching: 1/3
Marginal probability of winning, always switching: 2/3

Other marginal probabilities:

P(prize is behind Red door) = 1/3
P(you choose Red door) = 1/3, assuming you choose randomly.
P(you first choose the right door) = 1/3

The joint probability distribution gives us the means to answer many questions about random variables and their relationships.

Another Joint Distribution

Solar storm	Mike's battery on the fritz	Jan's radio works just fine	Mike's radio reception	Joint prob.
F	F	F	bad	0.0156
F	F	F	good	0.0468
F	F	F	none	0.0156
F	F	T	bad	0.1404
F	F	T	good	0.4212
F	F	T	none	0.1404
F	T	F	bad	0.0006
F	T	F	good	0.0002
F	T	F	none	0.0012
F	T	T	bad	0.0054
F	T	T	good	0.0018
F	T	T	none	0.0108

(continued)

Solar storm	Mike's battery on the fritz	Jan's radio works just fine	Mike's radio reception	Joint prob.
T	F	F	bad	0.0585
T	F	F	good	0.0117
T	F	F	none	0.0468
T	F	T	bad	0.039
T	F	T	good	0.0078
T	F	T	none	0.0312
T	T	F	bad	0.0006
T	T	F	good	0.00015
T	T	F	none	0.00225
T	T	T	bad	0.0004
T	T	T	good	0.0001
T	T	T	none	0.0015

Bayes Nets

A practical way to manage probabilistic inference when multiple variables (perhaps many) are involved.

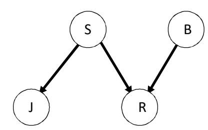
Requirement: The joint distribution is a "factored" distribution in which some random variables are either independent of or conditionally independent of most others.

Why Bayes Networks?

Reasoning about events involving many parts or contingencies generally requires that a joint probability distribution be known. Such a distribution might require thousands of parameters. Modeling at this level of detail is typically not practical.

Bayes Nets require making assumptions about the relevance of some conditions to others. Once the assumptions are made, the joint distribution can be "factored" so that there are many fewer separate parameters that must be specified.

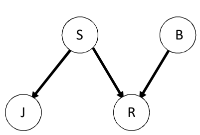
Bayes Net for the Radio Problem



S: [T, F] – "A solar storm is happening."
B: [T, F] – "Mike's battery is on the fritz."
J: [T, F] – "Jan's radio works just fine."
R: [bad, good, none] – "Mike's radio reception"

J is independent of B
J and R are conditionally independent, both conditioned on S.

Factored Distribution



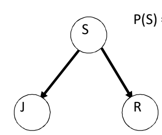
S: [P(T), P(F)] = [0.2, 0.8]
B: [P(T), P(F)] = [0.025, 0.975]
J: [P(T|S=T), P(F|S=T), P(T|S=F), P(F|S=F)] = [0.4, 0.6, 0.9, 0.1]
R: [P(bad|S=T,B=T), P(good|S=T,B=T), P(none|S=T,B=T), P(bad|S=T,B=F), …] = …

This factored distribution uses 20 parameters, rather than 24 for the unfactored version.

Not all of these are independent parameters: By using $\Sigma\, p_i = 1$, we can reduce the numbers to 12 and 23. For larger numbers of nodes, the savings are often much greater.

Working with the Bayes Net

P(S) = 0.2

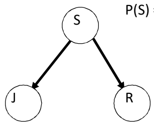


P(J|S) = 0.4
P(J|~S) = 0.9

P(R=bad|S) = 0.493
P(R=good|S) = 0.099
P(R=none|S) = 0.409
P(R=bad|~S) = 0.203
P(R=good|~S) = 0.588
P(R=none|~S) = 0.210

S: Solar Storm (A solar storm is happening.)
J: Jan's Radio (Jan's radio works just fine.)
R: Reception (Mike's radio's reception).

Forward Propagation (from causes to effects)



P(S) = 0.2

P(J|S) = 0.4
P(J|~S) = 0.9

P(R=bad|S) = 0.493
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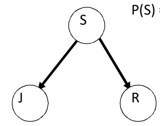
P(R=none|S) = 0.409
P(R=bad|~S) = 0.203
P(R=good|~S) = 0.588
P(R=none|~S) = 0.210

Suppose S: there is a solar storm.
Then P(J|S) is 0.4,   P(R=bad|S) = 0.493, etc.
Suppose ~S: no solar storm.
Then P(J|~S) is 0.9,   P(R=bad|~S) = 0.203, etc.
(These come directly from the given information.)

Marginal Probabilities (using forward propagation)



P(S) = 0.2

P(J|S) = 0.4
P(J|~S) = 0.9

P(R=bad|S) = 0.493
P(R=good|S) = 0.099
P(R=none|S) = 0.409
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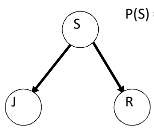
P(R=good|~S) = 0.588
P(R=none|~S) = 0.210

Then P(J), the probability that J is true in any situation, is
P(J) = P(J|S)P(S) + P(J|~S)(1-P(S)) = 0.08 + 0.72 = 0.8
And P(R=bad), the prob. that R is bad in any situation, is
P(R=bad) = P(R=bad|S)P(S) + P(R=bad|~S)(1-P(S)) =
= (0.493)(0.2) + (0.203)(0.8) = 0.261

Marginalizing means eliminating a contingency by summing the probabilities for its different cases (here S and ~S).

Using Bayes' Rule



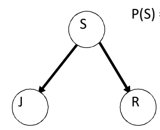
P(S) = 0.2

P(J|S) = 0.4
P(J|~S) = 0.9

P(R=bad|S) = 0.493
P(R=good|S) = 0.099
P(R=none|S) = 0.409
P(R=bad|~S) = 0.203
P(R=good|~S) = 0.588
P(R=none|~S) = 0.210

Suppose we know J: Jan's radio works just fine.
How do we update the probability of S?
Bayes' rule: P(S|J) = P(J|S)P(S)/P(J) = 0.08/0.8 = 0.1

Updating Probabilities of Consequences



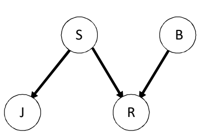
P(S) = 0.2

P(J|S) = 0.4
P(J|~S) = 0.9

P(R=bad|S) = 0.493
P(R=good|S) = 0.099
P(R=none|S) = 0.409
P(R=bad|~S) = 0.203
P(R=good|~S) = 0.588
P(R=none|~S) = 0.210
P(R=bad|J) ?

Suppose we know Jan's radio works just fine.
How do we update the probability of R=bad?
Use the revised probability of S:
P(R=bad|J) = P(R=bad|S)P(S|J) + P(R=bad|~S)P(~S|J) =
(0.493)(0.1) + (0.203)(0.9) = 0.049 + 0.183 = 0.232
Which is slightly lower than P(R=bad) = 0.261.

Handling Multiple Causes

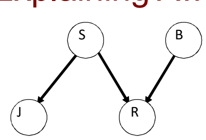


R	S	B	P(R|S,B)
R=bad	T	T	0.75
R=good	T	T	0.2
R=none	T	T	0.05
R=bad	T	F	0.4
R=good	T	F	0.5
R=none	T	F	0.1
R=bad	F	T	0.6
R=good	F	T	0.3
R=none	F	T	0.1
R=bad	F	F	0.2
R=good	F	F	0.2
R=none	F	F	0.6

B: [T, F] – "Mike's battery is on the fritz."

P(B) = 0.025

Explaining Away



R	S	B	P(R|S,B)
R=bad	T	T	0.75
R=good	T	T	0.2
R=none	T	T	0.05
R=bad	T	F	0.4
R=good	T	F	0.5
R=none	T	F	0.1
R=bad	F	T	0.6
R=good	F	T	0.3
R=none	F	T	0.1
R=bad	F	F	0.2
R=good	F	F	0.2
R=none	F	F	0.6

Suppose R=bad. This raises the probability for each cause:
P(S|R=bad) = 0.378, P(B|R=bad) = P(R=bad|B) P(B)/
P(R=bad) = 0.027

Now, in addition, suppose not J (Jan's radio not fine).
Not J makes it more likely that S is true,
"And this explains R=bad."
B is now less probable: P(B|R=bad, J=F) = 0.016.

Jd Joint Distribution

Benefits of Bayes Nets

"The decomposition of large probabilistic domains into weakly connected subsets through conditional independence is one of the most important developments in the recent history of AI."

(Russell & Norvig, 3e. p499.)

Jd Joint Distribution

Benefits of Bayes Nets

The joint probability distribution with boolean random variables normally requires $2^n - 1$ independent parameters.

With Bayes Nets we only specify these parameters:

1. "root" node probabilities.
   e. g., P(A=true) = 0.2; P(A=false)=0.8.

2. For each non-root node, a table of 2^k values, where k is the number of parents of that node.
   Typically $k < 5$.

3. Propagating probabilities happens along the paths in the net.
   With a full joint prob. dist., many more computations may be needed.