Uncertainty in AI: Probabilistic Reasoning

CSE 415: Introduction to Artificial Intelligence
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Motivation

Logical reasoning has limitations:

It requires that assumptions be considered “certain”.

It typically uses general rules. General rules that are reliable may be difficult to come by.

Logical reasoning can be awkward for certain structured domains such as time and space.

Bayes' Rule

E: Some evidence exists, i.e., a particular condition is true
H: some hypothesis is true.

P(E|H) = probability of E given H.
P(E|~H) = probability of E given not H.
P(H) = probability of H, independent of E.

\[
P(E|H) \frac{P(H)}{P(E)} = \frac{P(E|H) P(H)}{P(E)}
\]

P(E) = P(E|H) P(H) + P(E|~H)(1 - P(H))

Generalizing Modus Ponens

Modus Ponens:
P → Q
P
Q

Bayes' Rule: (general idea)

If P then sometimes Q
P
Maybe Q

(Bayes' rule lets us calculate the probability of Q, taking P into account.)

Applying Bayes’ Rule

E: The patient's white blood cell count exceeds 110% of average.
H: The patient is infected with tetanus.

P[E|H] = 0.8
P[E|~H] = 0.3
P(H) = 0.01
P(E) = P[E|H] P(H) + P[E|~H](1 - P(H))

\[
P(E|H) \frac{P(H)}{P(E)} = \frac{P(E|H) P(H)}{P(E)}
\]

P(E|H) P(H) 0.008
P(E)

P[H|E] = = = = = = = = = 0.0262
(0.8)(0.01) + (0.3)(0.99) 0.305

P[E] = P[E|H] P(H) + P[E|~H](1 - P(H))
Odds

Odds are 10 to 1 it will rain tomorrow.

\[ \frac{P(\text{rain})}{P(\neg \text{rain})} = \frac{10}{11} \]

Suppose \( P(A) = \frac{1}{4} \)
Then \( \text{O}(A) = \frac{P(A)}{P(\neg A)} = \frac{1}{3} \)

in general:
\[ \text{O}(A) = \frac{P(A)}{1 - P(A)} \]

Bayes’ Rule reformulated...

\[ \frac{P(\text{E} | \text{H})}{P(\text{E})} \]

\[ \frac{P(\text{H} | \text{E})}{P(\text{E})} \]

\[ \frac{P(\text{E} | \neg \text{H})}{P(\text{E})} \]

Odds-Likelihood

Form of Bayes’ Rule

E: The patient’s white blood cell count exceeds 110% of average.

H: The patient is infected with tetanus.

\[ \text{O}(\text{H}) = 0.01/0.99 = \lambda \]

\[ \text{O}(\text{H} | \text{E}) = \lambda \text{O}(\text{H}) \]

\[ \text{O}(\text{H} | \neg \text{E}) = \lambda' \text{O}(\text{H}) \]

\[ \lambda = P(\text{E} | \text{H})/P(\neg \text{E} | \neg \text{H}) \approx 2.67 \]

\[ \lambda' = P(\text{E} | \neg \text{H})/P(\neg \text{E} | \text{H}) \approx 0.286 \]

Total

A random variable \( X \) is a symbol that represents a class of events that may occur any number of times, and which take on values in a given set \( D \) called a domain.

Example: Let \( C \) be a coin-toss random variable with domain \( D=\{H, T\} \).

A probability distribution \( P \) for a random variable is a function that assigns to each domain element \( d_i \) a value \( p_i \) in the range 0 to 1. If \( D \) is finite then \( P \) is often given as a table.

Example with \( n=2 \):

<table>
<thead>
<tr>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( P(D_1, D_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>rain</td>
<td>no crash</td>
<td>1/4</td>
</tr>
<tr>
<td>rain</td>
<td>crash</td>
<td>1/8</td>
</tr>
<tr>
<td>clear</td>
<td>no crash</td>
<td>5/16</td>
</tr>
<tr>
<td>clear</td>
<td>crash</td>
<td>1/16</td>
</tr>
<tr>
<td>snow</td>
<td>no crash</td>
<td>1/8</td>
</tr>
<tr>
<td>snow</td>
<td>crash</td>
<td>1/8</td>
</tr>
</tbody>
</table>

The conditional probability of \( P(X=d | Y=d) \) is the probability of \( X=d \), given \( Y=d \).

Example:

Let \( D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \).

Let \( \text{Odd} \) be the set of outcomes \( X=1, X=3, ..., X=9 \).

Let \( \text{Even} \) be the set of outcomes \( X=0, X=2, ..., X=8 \).

Let \( \text{Prime} \) be the set of outcomes \( X=2, X=3, X=5, X=7 \).

\( P(\text{Odd}) = 0.5 \)

\( P(\text{Prime}) = 0.4 \)

\( P(\text{Prime} | \text{Odd}) = 0.75 \)

\( P(\text{Prime} | \neg \text{Odd}) = 0.6 \).
Laws of Probability

The sum rule: \( \sum_{i=1}^{m} P(X = d_i) = 1 \) Adding the probabilities of all the possible outcomes for a random variable must give a total of 1.0.

The product rule: \( P(x) P(y | x) = P(x, y) \).

The Monty Hall Problem

There are three doors: a red door, green door, and blue door. Behind one is a car, and behind the other two are goats. You get to keep whatever is behind the door you choose.

You choose a door (say, red).

The host opens one of the other doors (say, green), which reveals a goat.

The host says, "Would you like to select the OTHER door?"

Should you switch?