The α-β algorithm

function **Alpha-Beta-Search**(state) returns an action
inputs: state, current state in game
\[ v \leftarrow \text{Max-Value}(state, -\infty, +\infty) \]
return the action in **Successors**(state) with value \( v \)

function **Max-Value**(state, \( \alpha, \beta \)) returns a utility value
inputs: state, current state in game
\[ \alpha, \text{the value of the best alternative for } \text{Max} \text{ along the path to state} \]
\[ \beta, \text{the value of the best alternative for } \text{Min} \text{ along the path to state} \]
if **Terminal-Test**(state) then return **Utility**(state)
\[ v \leftarrow -\infty \]
for \( a, s \) in **Successors**(state) do
\[ v \leftarrow \text{Max}(v, \text{Min-Value}(s, \alpha, \beta)) \]
\[ \text{if } v \geq \beta \text{ then return } v \]
\[ \alpha \leftarrow \text{Max}(\alpha, v) \]
return \( v \)
The $\alpha$-$\beta$ algorithm

```
function Min-Value(state, $\alpha$, $\beta$) returns a utility value
    inputs: state, current state in game
            $\alpha$, the value of the best alternative for MAX along the path to state
            $\beta$, the value of the best alternative for MIN along the path to state
    if Terminal-Test(state) then return Utility(state)
    $v \leftarrow +\infty$
    for $a$, $s$ in Successors(state) do
        $v \leftarrow \min(v, \text{Max-Value}(s, \alpha, \beta))$
        if $v \leq \alpha$ then return $v$
        $\beta \leftarrow \min(\beta, v)$
    return $v$
```

Should $\alpha$ and $\beta$ be passed by value or reference? ie. Should a lower $\alpha$ affect an upper one?
Alpha-Beta Pruning Example

α is MAX’s best alternative here or above
β is MIN’s best alternative here or above