## The $\alpha-\beta$ algorithm

function Alpha-Beta-Search(state) returns an action inputs: state, current state in game
$v \leftarrow \operatorname{Max}-\operatorname{ValuE}($ state $,-\infty,+\infty)$
return the action in SUCCESSORS(state) with value $v$
function Max-Value(state, $\alpha, \beta$ ) returns a utility value
inputs: state, current state in game
$\alpha$, the value of the best alternative for MAX along the path to state
$\beta$, the value of the best alternative for MIN along the path to state
if Terminal-Test(state) then return Utility(state)
$v \leftarrow-\infty$
for $a, s$ in SUCCESSORS(state) do

$$
\begin{aligned}
& v \leftarrow \operatorname{Max}(v, \operatorname{Min}-\operatorname{VALUE}(s, \alpha, \beta)) \\
& \text { if } v \geq \beta \text { then return } v \quad \text { Cutoff }
\end{aligned}
$$

$$
\alpha \leftarrow \operatorname{MAX}(\alpha, v)
$$

return $v$


## The $\alpha-\beta$ algorithm

function Min-Value(state, $\alpha, \beta$ ) returns a utility value inputs: state, current state in game $\alpha$, the value of the best alternative for MAX along the path to state $\beta$, the value of the best alternative for MIN along the path to state
if Terminal-Test(state) then return Utility (state)
$v \leftarrow+\infty$
for $a, s$ in SUCCESSORS(state) do
$v \leftarrow \operatorname{Min}(v, \operatorname{Max}-\operatorname{Value}(s, \alpha, \beta))$
if $v \leq \alpha$ then return $v \quad$ Cutoff
$\beta \leftarrow \operatorname{Min}(\beta, v)$
return $v$

Should $\alpha$ and $\beta$ be passed by value or reference? ie. Should a lower a affect an upper one?

## Alpha-Beta Pruning Example



