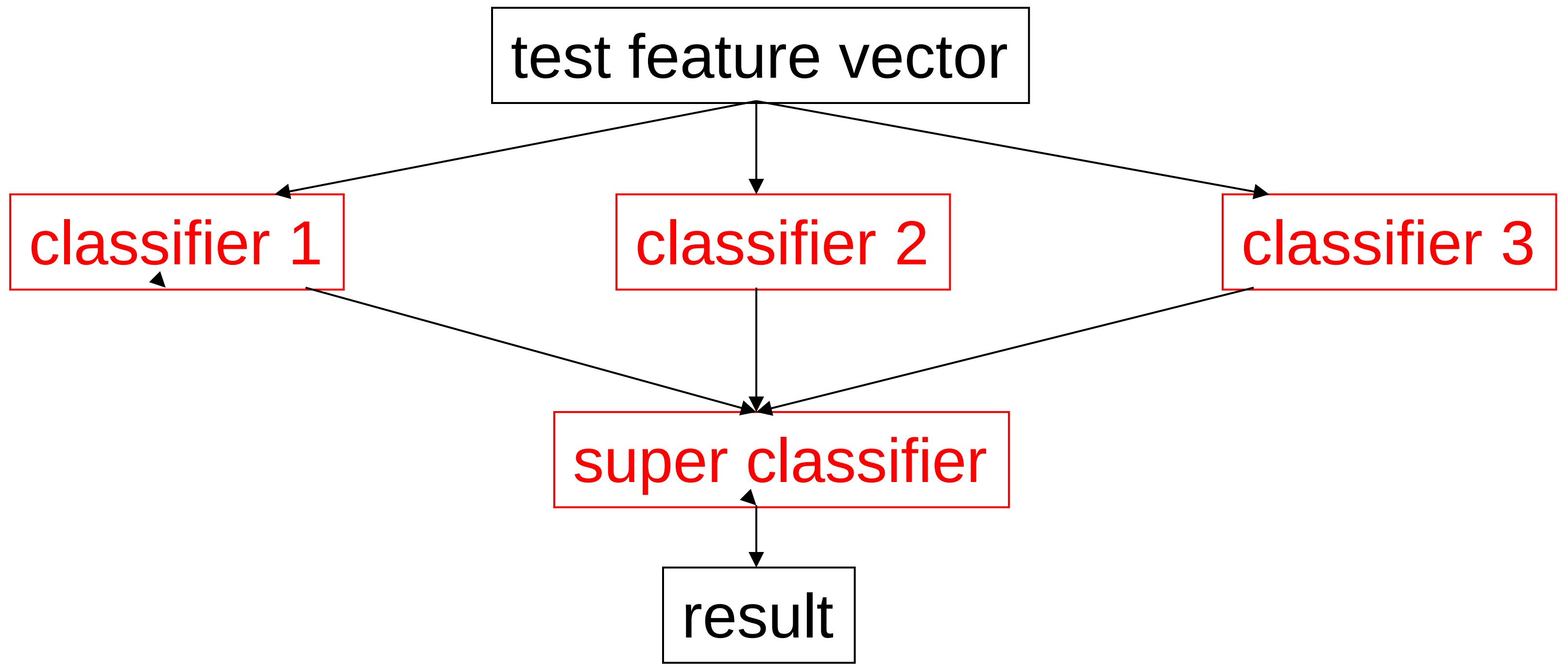
More Learning

- Ensembles
- Bayes Rule
- Neural Nets
- K-means Clustering
- EM Clustering
- WEKA

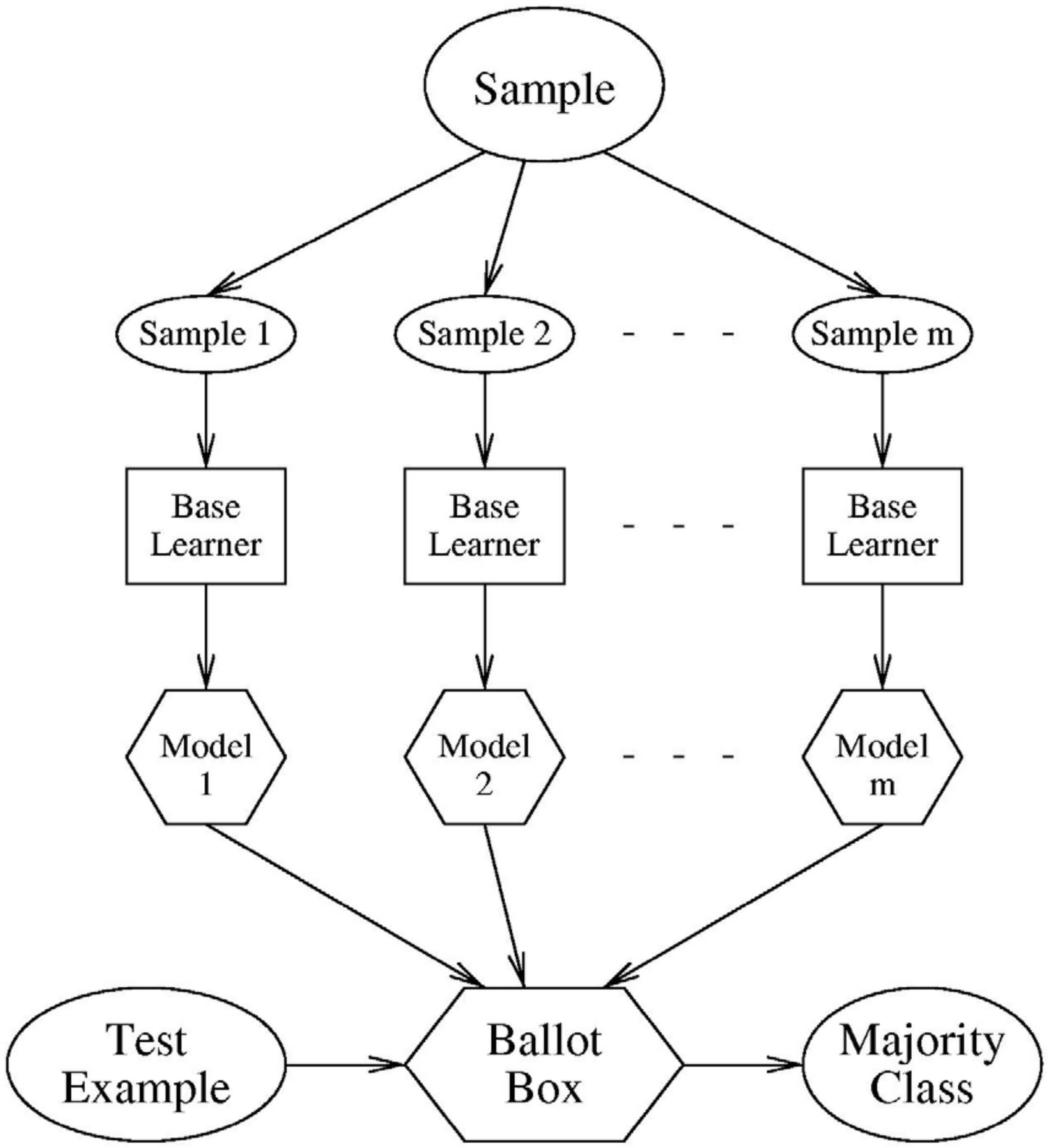
Ensembles

- An ensemble is a set of classifiers whose combined results give the final decision.



Bagging

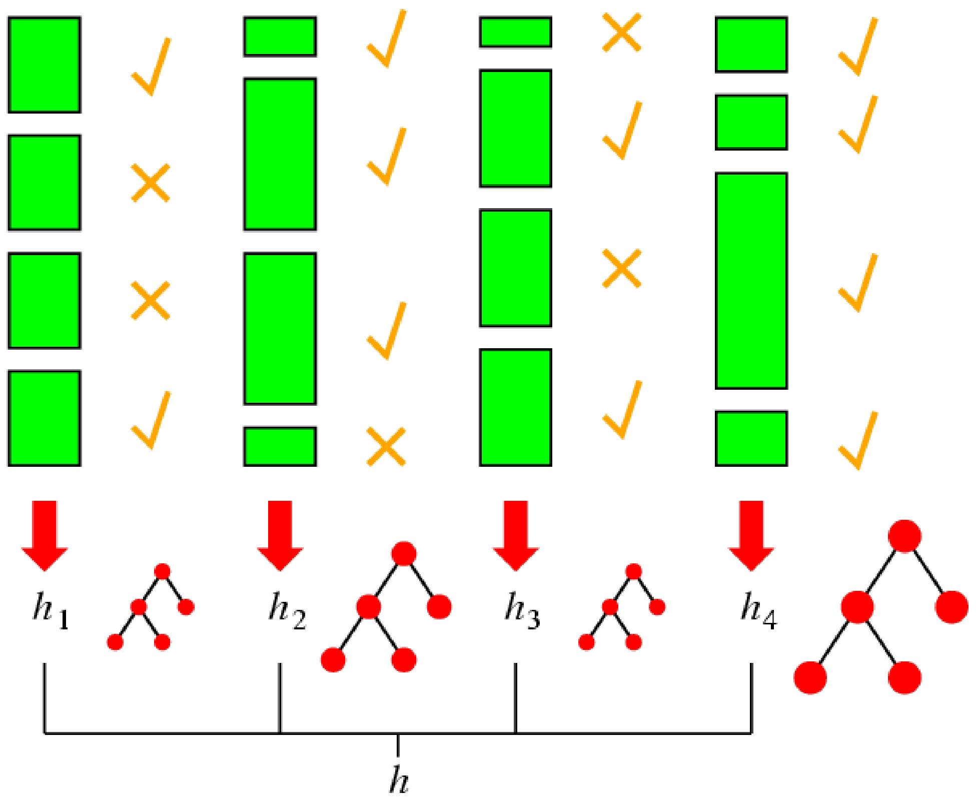
- Generate "bootstrap" replicates of training set by sampling with replacement
- Learn one model on each replicate
- Combine by uniform voting


Sample
Sample 1
Sample 2
- - - -
Sample m
Base Learner
Base Learner
Base Learner
Model 1
Model 2
Model m
Test Example
Ballot Box
Majority Class

Boosting

- Maintain vector of weights for examples
- Initialize with uniform weights
- Loop:
  - Apply learner to weighted examples (or sample)
  - Increase weights of misclassified examples
- Combine models by weighted voting

Idea of Boosting

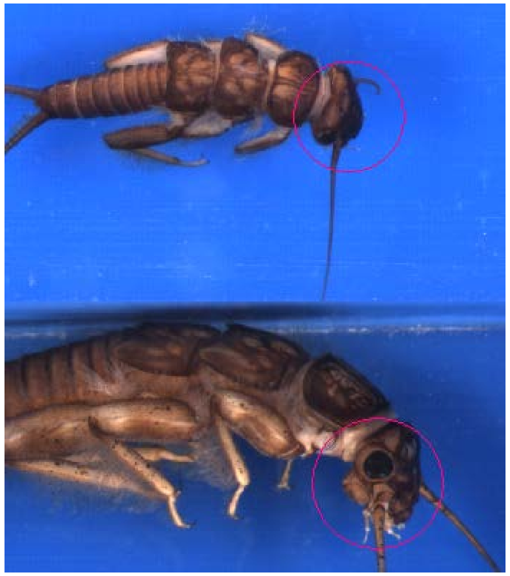


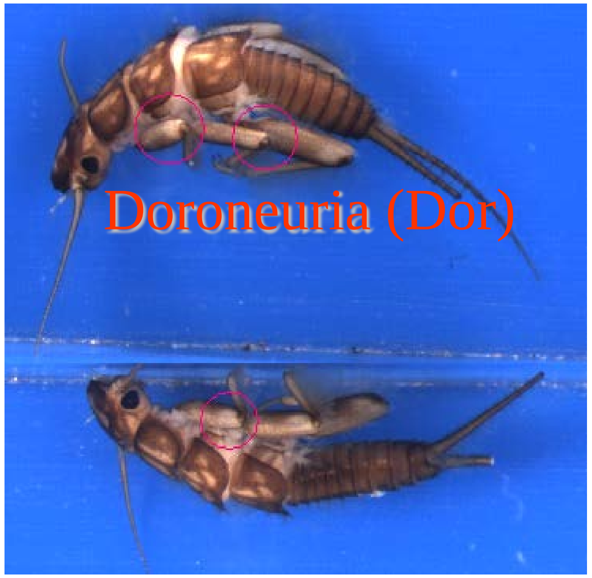
ADABoost

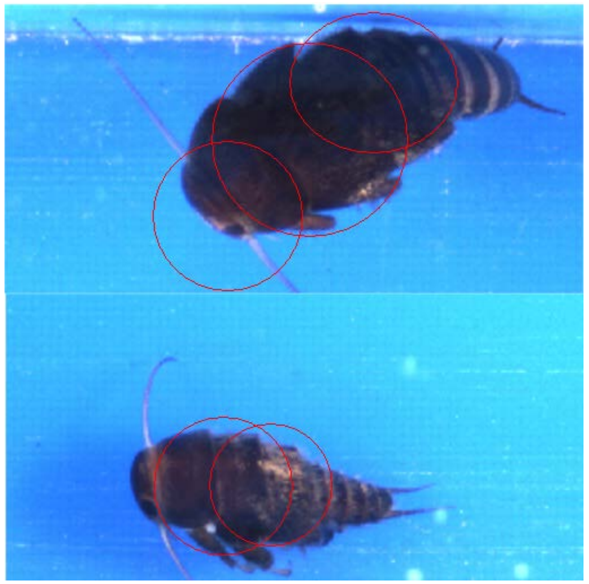
- ADABoost boosts the accuracy of the original learning algorithm.

- If the original learning algorithm does slightly better than 50% accuracy, ADABoost with a large enough number of classifiers is guaranteed to classify the training data perfectly.

Sample Application: Insect Recognition







Using circular regions of interest selected by an interest operator, train a classifier to recognize the different classes of insects.

Boosting Comparison

- **ADTree classifier only** **(alternating decision tree)**

• Correctly Classified Instances	268	70.1571 %
• Incorrectly Classified Instances	114	29.8429 %
• Mean absolute error	0.3855	
• Relative absolute error	77.2229 %	

Classified as ->	Hesperperla	Doroneuria
Real Hesperperlas	167	28
Real Doroneuria	51	136

Boosting Comparison

AdaboostM1 with ADTree classifier

• Correctly Classified Instances	303	79.3194 %
• Incorrectly Classified Instances	79	20.6806 %
• Mean absolute error	0.2277	
• Relative absolute error	45.6144 %	

Classified as ->	Hesperperla	Doroneuria
Real Hesperperlas	167	28
Real Doroneuria	51	136

Boosting Comparison

- **RepTree classifier only** **(reduced error pruning)**

• Correctly Classified Instances	294	75.3846 %
• Incorrectly Classified Instances	96	24.6154 %
• Mean absolute error	0.3012	
• Relative absolute error	60.606 %	

Classified as ->	Hesperperla	Doroneuria
Real Hesperperlas	169	41
Real Doroneuria	55	125

Boosting Comparison

AdaboostM1 with RepTree classifier

• Correctly Classified Instances	324	83.0769 %
• Incorrectly Classified Instances	66	16.9231 %
• Mean absolute error	0.1978	
• Relative absolute error	39.7848 %	

Classified as ->	Hesperperla	Doroneuria
Real Hesperperlas	180	30
Real Doroneuria	36	144

Bayesian Learning

- Bayes' Rule provides a way to calculate probability of a hypothesis based on
  - its prior probability
  - the probability of observing the data, given that hypothesis
  - the observed data (feature vector)

Bayes' Rule

$$\boxed{P(h \mid X) = \frac{P(X \mid h)\; P(h)}{P(X)}}$$

$\leftarrow$ Often assumed constant and left out.

- h is the hypothesis (such as the class).
- X is the feature vector to be classified.
- P(X | h) is the prior probability that this feature vector occurs, given that h is true.
- P(h) is the prior probability of hypothesis h.
- P(X) = the prior probability of the feature vector X.
- These priors are usually calculated from frequencies in the training data set.

Example

x1	x2	x3	y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

- Suppose we want to know the probabilty of class 1 for feature vector [0,1,0].

- $P(1 \mid [0,1,0]) = P([0,1,0] \mid 1)\ P(1)\ /\ P([0,1,0])$
  $= (0.25)\ (0.5)\ /\ (.125)$
  $= 1.0$

Of course the training set would be much bigger and for real data could include multiple instances of a given feature vector.

MAP

- Suppose H is a set of candidate hypotheses.
- We would like to find the most probable h in H.
- h_{MAP} is a MAP (maxiumum a posteriori) hypothesis if

$$h_{MAP} = \underset{h \,\varepsilon\, H}{\operatorname{argmax}} \; P(h \mid X)$$

- This just says to calculate P(h | X) by Bayes' rule for each possible class h and take the one that gets the highest score.

Cancer Test Example

P(cancer) = .008
P(not cancer) = .992
P(positive | cancer) = .98
P(positive | not cancer) = .03
P(negative | cancer) = .02
P(negative | not cancer) =.97

Priors

New patient's test comes back positive.

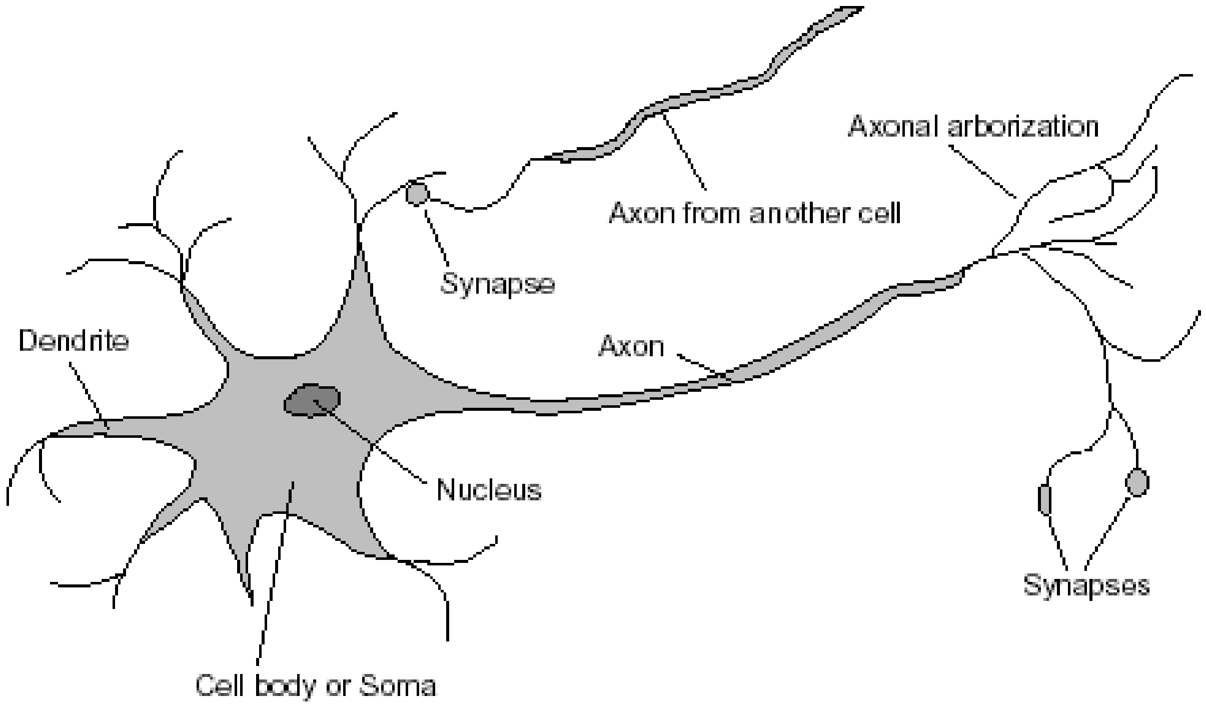
P(cancer | positive) = P(positive | cancer) P(cancer)
= (.98) (.008) = .0078
P(not cancer | positive = P(positive | not cancer) P(not cancer)
= (.03) (.992) = .0298
h_{MAP} would say it's not cancer. Depends strongly on priors!

Neural Net Learning

- Motivated by studies of the brain.
- A network of “artificial neurons” that learns a function.
- Doesn’t have clear decision rules like decision trees, but highly successful in many different applications. (e.g. face detection)
- Our hierarchical classifier used neural net classifiers as its components.

Brains

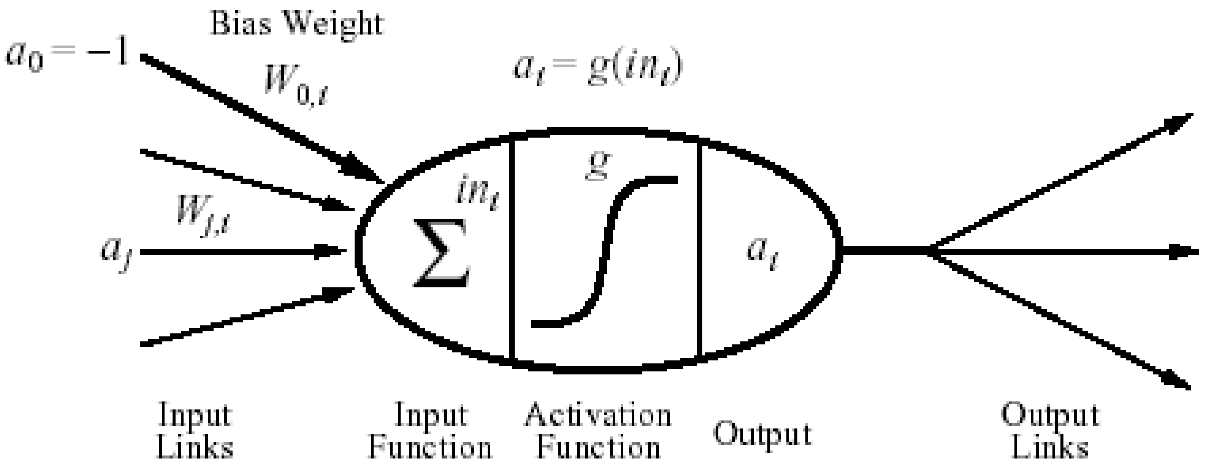
10^{11} neurons of > 20 types, 10^{14} synapses, 1ms–10ms cycle time
Signals are noisy "spike trains" of electrical potential



McCulloch–Pitts "unit"

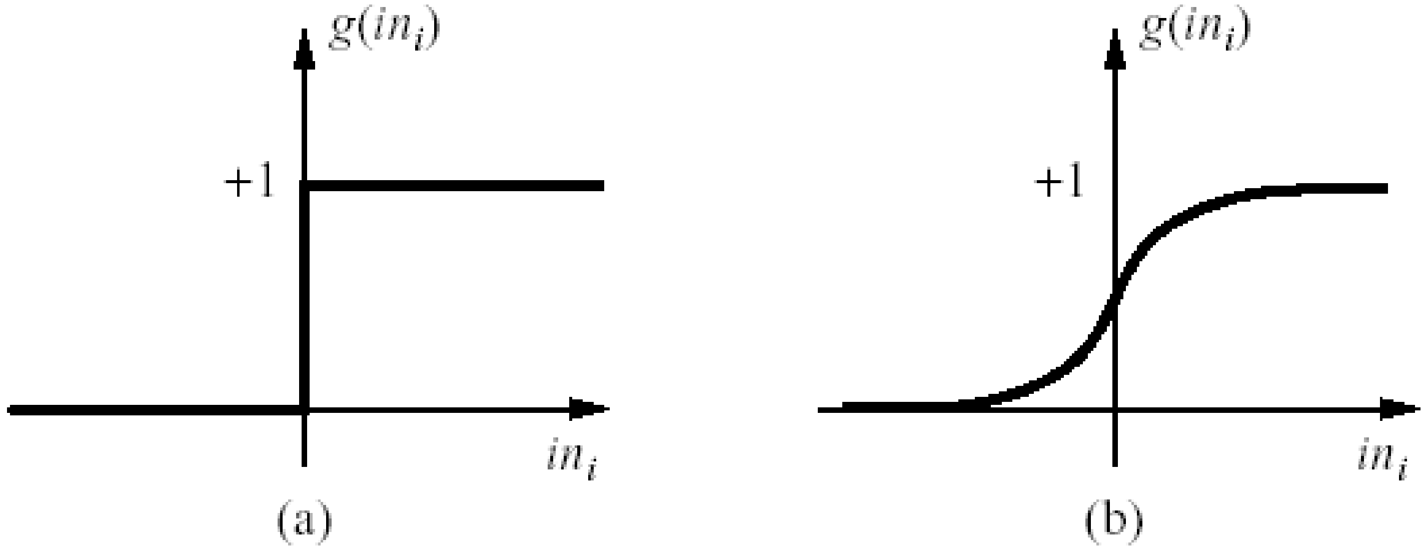
Output is a "squashed" linear function of the inputs:

$$a_i \leftarrow g(in_i) = g\left(\Sigma_j W_{j,i} a_j\right)$$



A gross oversimplification of real neurons, but its purpose is
to develop understanding of what networks of simple units can do

Activation functions

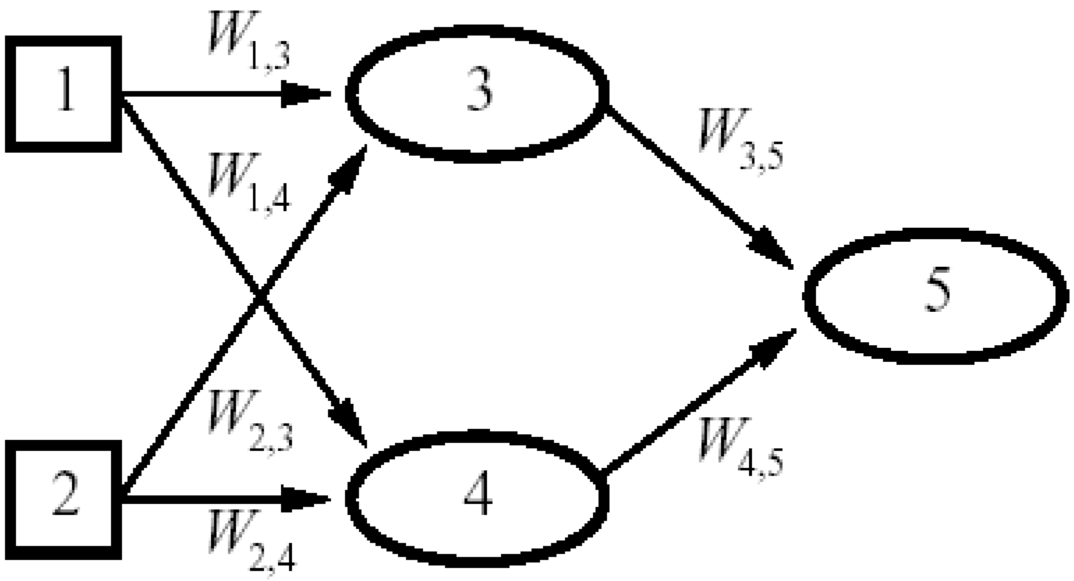


(a) is a step function or threshold function

(b) is a sigmoid function $1/(1+e^{-x})$

Changing the bias weight $W_{0,i}$ moves the threshold location

Feed-forward example



Feed-forward network = a parameterized family of nonlinear functions:

$$\begin{aligned} a_5 &= g(W_{3,5} \cdot a_3 + W_{4,5} \cdot a_4) \\ &= g(W_{3,5} \cdot g(W_{1,3} \cdot a_1 + W_{2,3} \cdot a_2) + W_{4,5} \cdot g(W_{1,4} \cdot a_1 + W_{2,4} \cdot a_2)) \end{aligned}$$

Adjusting weights changes the function: do learning this way!

Perceptron learning

Learn by adjusting weights to reduce error on training set

The squared error for an example with input $\mathbf{x}$ and true output y is

$$E = \frac{1}{2}Err^2 \equiv \frac{1}{2}(y - h_{\mathbf{W}}(\mathbf{x}))^2 \; ,$$

Perform optimization search by gradient descent:

$$\begin{aligned}\frac{\partial E}{\partial W_j} &= Err \times \frac{\partial Err}{\partial W_j} = Err \times \frac{\partial}{\partial W_j}\left(y - g(\sum_{j=0}^{n} W_j x_j)\right) \\ &= -Err \times g'(in) \times x_j\end{aligned}$$

Simple weight update rule:

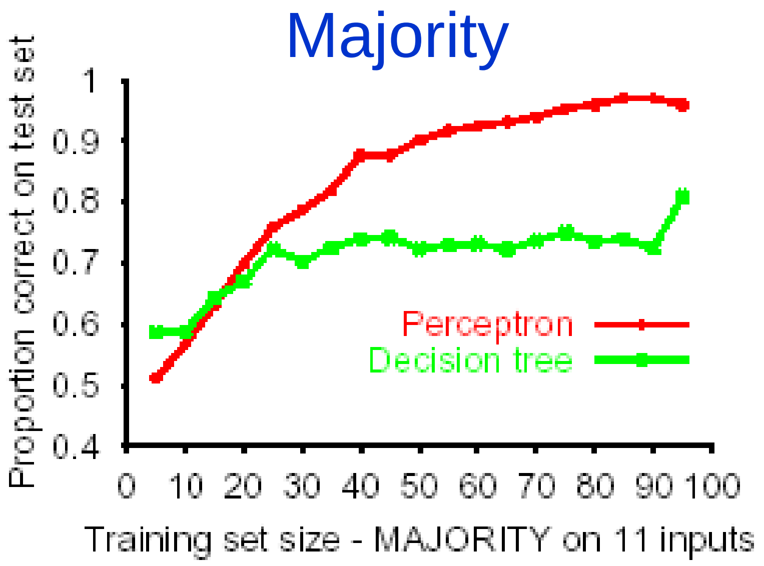
$$W_j \leftarrow W_j + \alpha \times Err \times g'(in) \times x_j$$

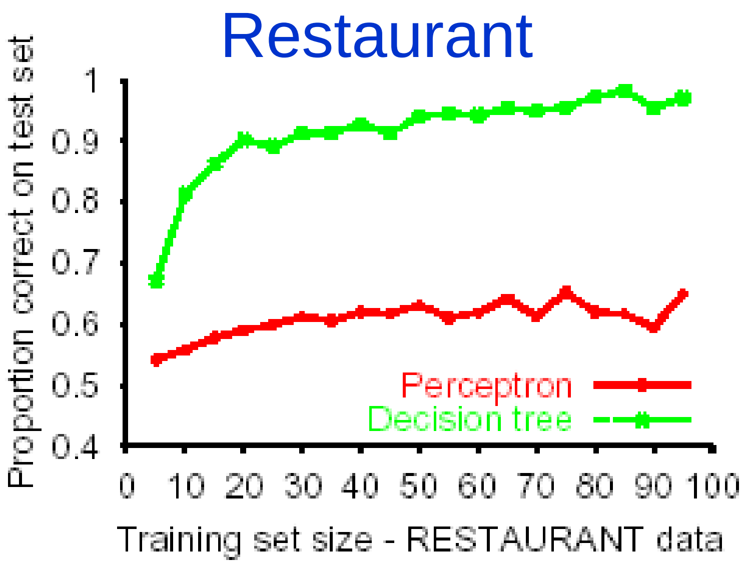
E.g., +ve error $\Rightarrow$ increase network output

$\Rightarrow$ increase weights on +ve inputs, decrease on -ve inputs

Perceptron learning contd.

Perceptron learning rule converges to a consistent function
for any linearly separable data set



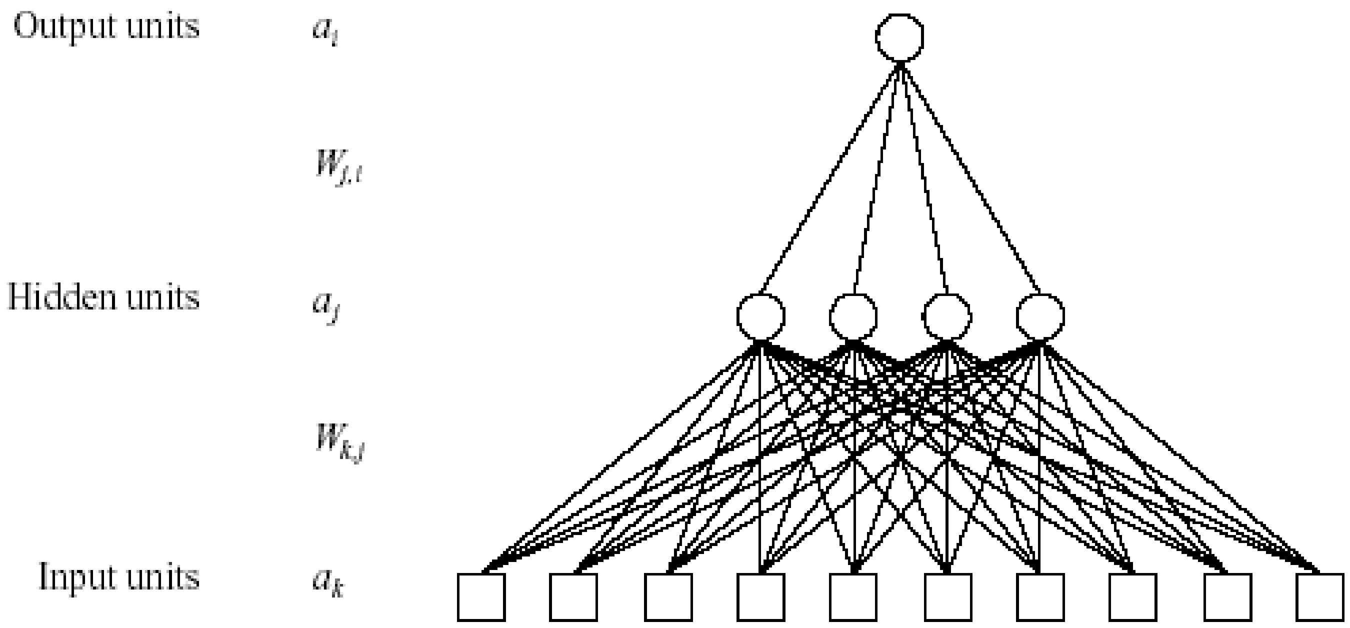


Perceptron learns majority function easily, DTL is hopeless

DTL learns restaurant function easily, perceptron cannot represent it

Multilayer perceptrons

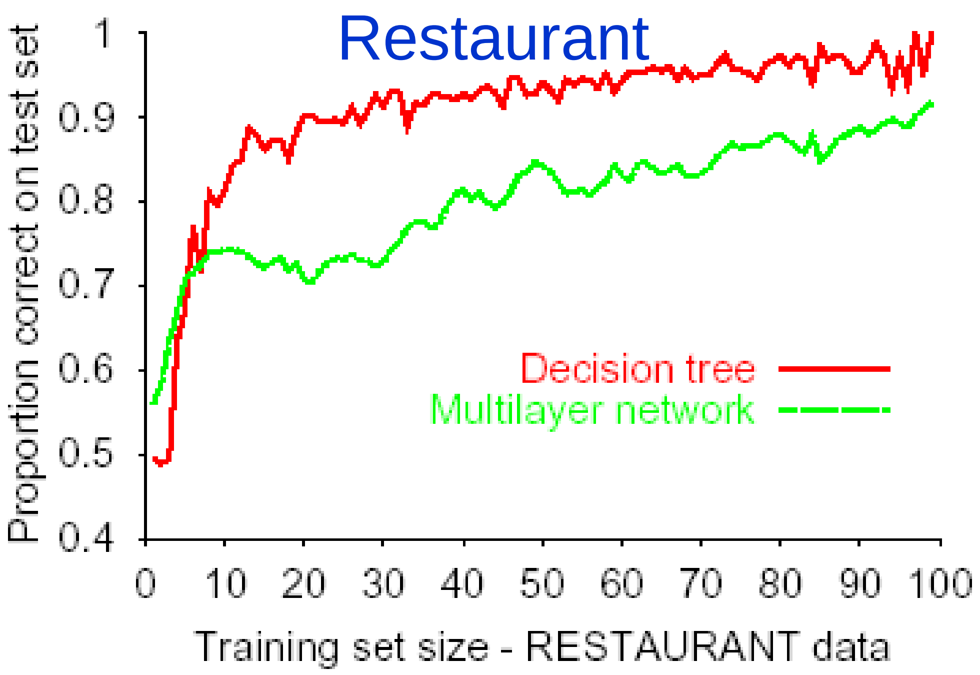
Layers are usually fully connected;
numbers of hidden units typically chosen by hand



Multilayer perceptrons with back-propagation learning are more powerful.

Back-propagation learning contd.

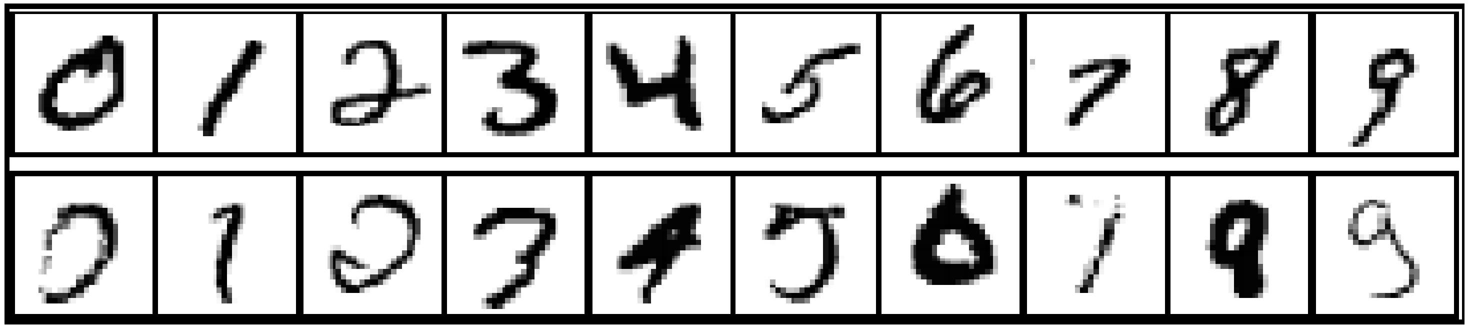
Learning curve for MLP with 4 hidden units:



MLPs are quite good for complex pattern recognition tasks,
but resulting hypotheses cannot be understood easily

Decision tree still wins, but not by as much.

Handwritten digit recognition



3-nearest-neighbor = 2.4% error
400–300–10 unit MLP = 1.6% error
LeNet: 768–192–30–10 unit MLP = 0.9% error

Current best (kernel machines, vision algorithms) ≈ 0.6% error

Neural nets (MLP) work great on handwritten digit recognition.

Kernel Machines

- A relatively new learning methodology (1992) derived from statistical learning theory.
- Became famous when it gave accuracy comparable to neural nets in a handwriting recognition class.
- Was introduced to computer vision researchers by Tomaso Poggio at MIT who started using it for face detection and got better results than neural nets.
- Has become very popular and widely used with packages available.

Support Vector Machines (SVM)

- Support vector machines are learning algorithms that try to find a hyperplane that separates the different classes of data the most.

- They are a specific kind of kernel machines based on two key ideas:

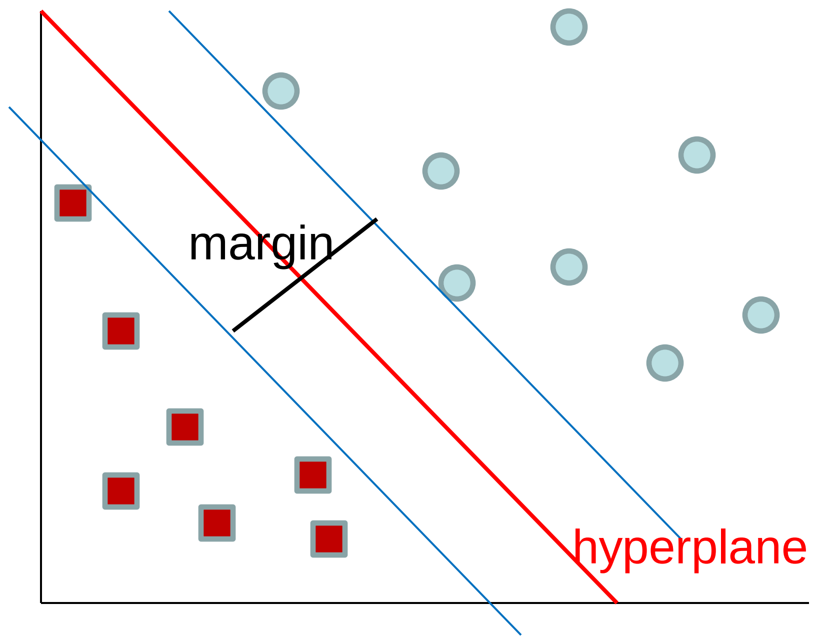
  - maximum margin hyperplanes

  - a kernel ‘trick’

Maximal Margin (2 class problem)

In 2D space,
a hyperplane is
a line.

In 3D space,
it is a plane.



Find the hyperplane with maximal margin for all the points. This originates an optimization problem which has a unique solution.

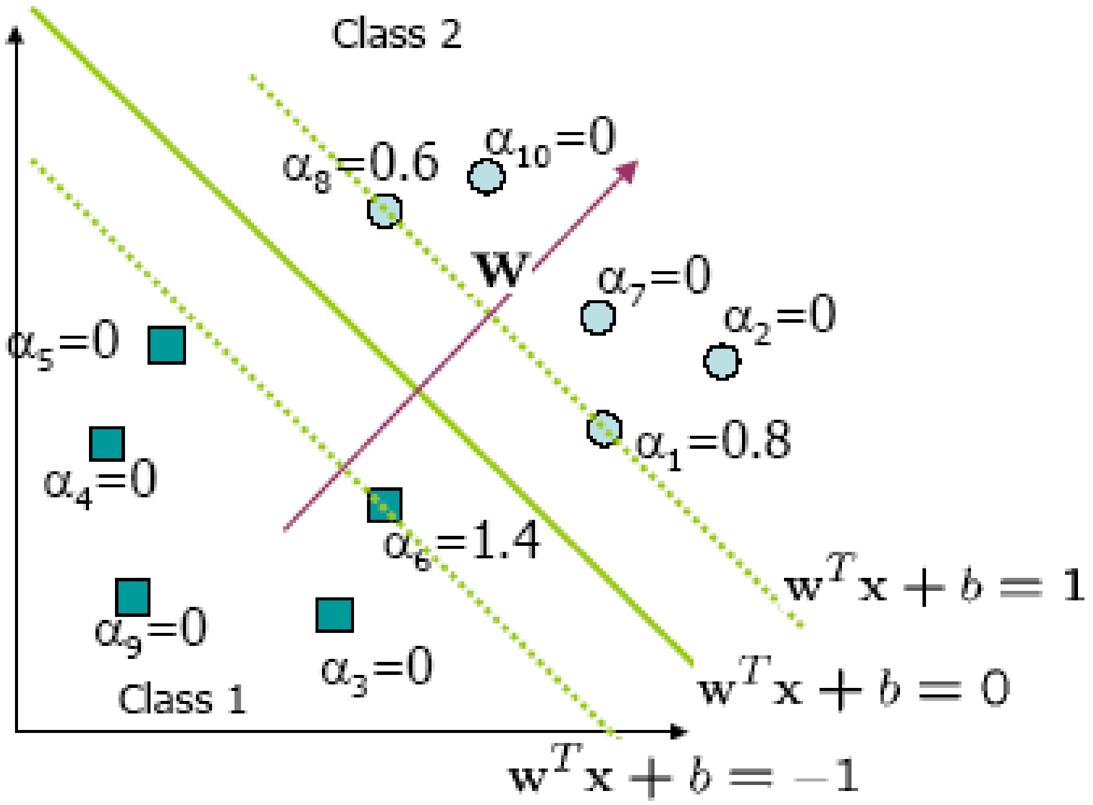
Support Vectors

- The weights α_i associated with data points are zero, except for those points closest to the separator.

- The points with nonzero weights are called the support vectors (because they hold up the separating plane).

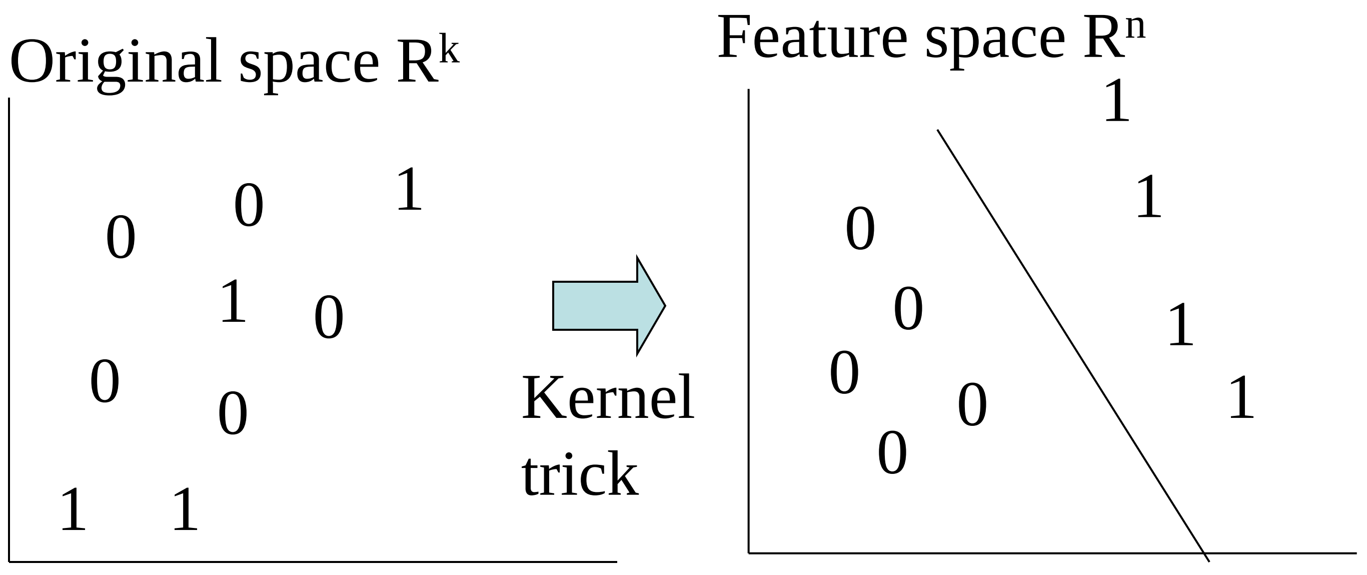
- Because there are many fewer support vectors than total data points, the number of parameters defining the optimal separator is small.

A Geometric Interpretation

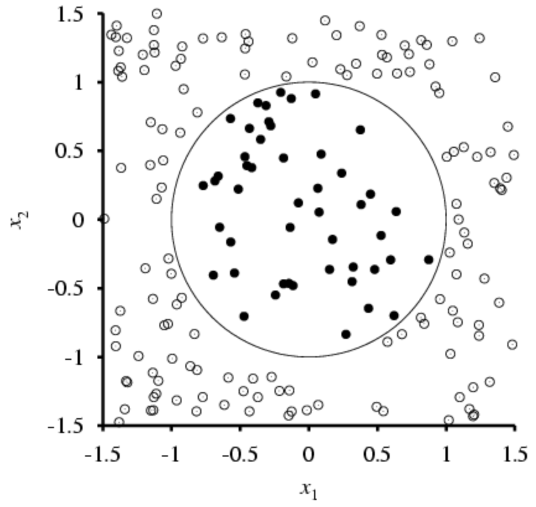


The Kernel Trick

The SVM algorithm implicitly maps the original data to a feature space of possibly infinite dimension in which data (which is not separable in the original space) becomes separable in the feature space.



Example from Text



True decision boundary is $x_1^2 + x_2^2 \leq 1$.

- Mapping the data to the 3D space defined by $f_1 = x_1^2$, $f_2 = x_2^2$, $f_3 = 2^{1/2} x_1 x_2$ makes it linearly separable by a plane in 3D.

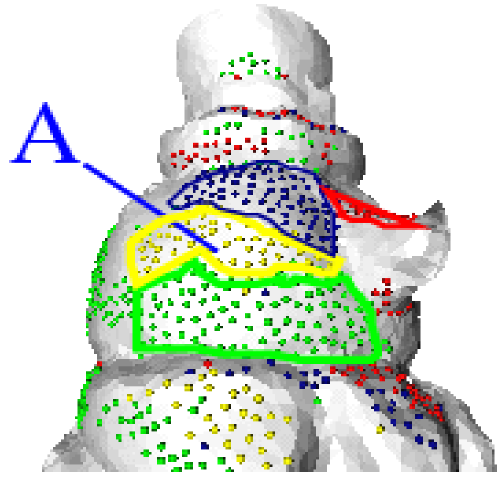
- For this problem $F(x_i) \bullet F(x_j)$ is just $(xi \bullet xj)^{2,}$ which is called a kernel function.

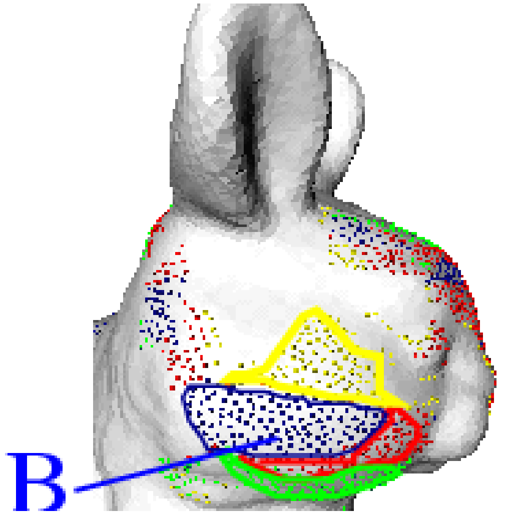
Kernel Functions

- The kernel function is designed by the developer of the SVM.
- It is applied to pairs of input data to evaluate dot products in some corresponding feature space.
- Kernels can be all sorts of functions including polynomials and exponentials.

Kernel Function used in our 3D Computer Vision Work

- $k(A,B) = \exp(-\theta^2_{AB}/\sigma^2)$
- A and B are shape descriptors (big vectors).
- θ is the angle between these vectors.
- σ^2 is the "width" of the kernel.





Unsupervised Learning

- Find patterns in the data.
- Group the data into clusters.
- Many clustering algorithms.
  - K means clustering
  - EM clustering
  - Graph-Theoretic Clustering
  - Clustering by Graph Cuts
  - etc

Clustering by K-means Algorithm

Form K-means clusters from a set of n-dimensional feature vectors

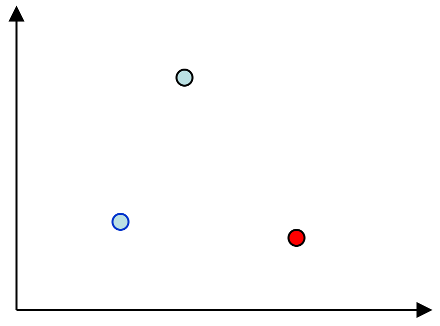
1. Set ic (iteration count) to 1

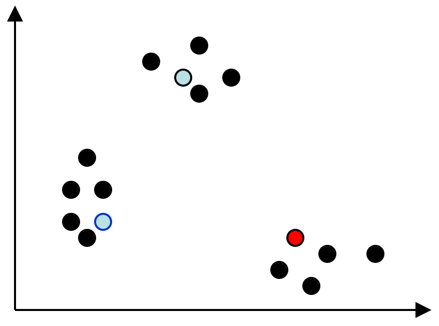
2. Choose randomly a set of K means $m_1(1), \ldots, m_K(1)$.

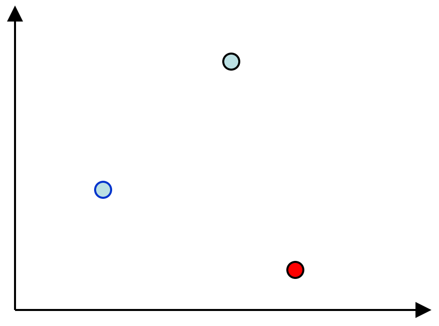
3. For each vector x_i, compute $D(x_i, m_k(ic))$, $k=1,\ldots K$ and assign x_i to the cluster C_j with nearest mean.

4. Increment ic by 1, update the means to get $m_1(ic),\ldots,m_K(ic)$.

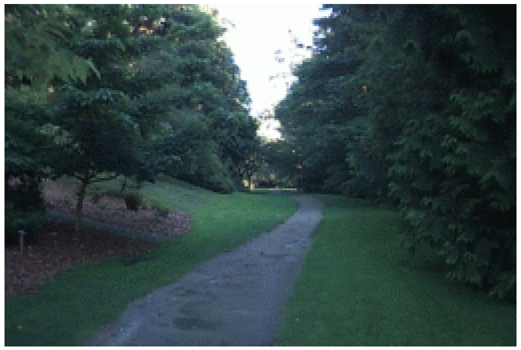
5. Repeat steps 3 and 4 until $C_k(ic) = C_k(ic+1)$ for all k.



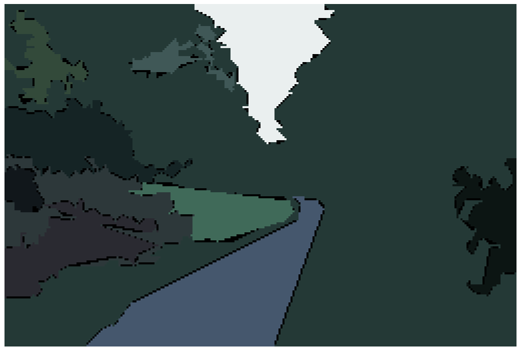




K-Means Classifier (shown on RGB color data)



original data
one RGB per pixel



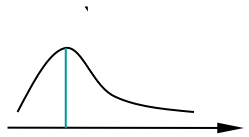
color clusters

K-Means → EM

The clusters are usually Gaussian distributions.

- Boot Step:
  - Initialize K clusters: $C_1, \ldots, C_K$

    (μ_j, Σ_j) and $P(C_j)$ for each cluster j.



- Iteration Step:
  - Estimate the cluster of each datum

    $p(C_j \mid x_i)$ ⇨ Expectation

  - Re-estimate the cluster parameters

    $(\mu_j, \Sigma_j), p(C_j)$ For each cluster j ⇨ Maximization

The resultant set of clusters is called a **mixture model**;
if the distributions are Gaussian, it's a Gaussian mixture.

EM Algorithm Summary

- Boot Step:
  - Initialize *K* clusters: C$_1$, …, C_K

    (μ_j, Σ_j) and $p(C_j)$ for each cluster *j*.
- Iteration Step:
  - Expectation Step

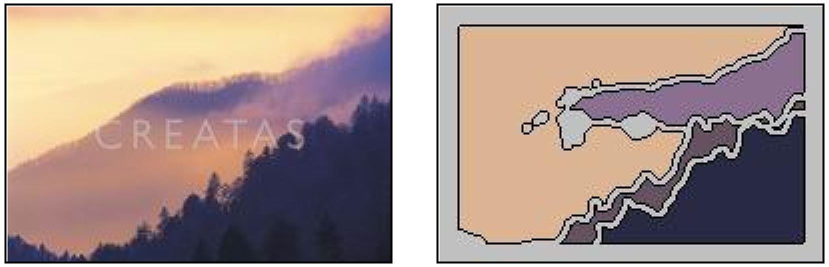
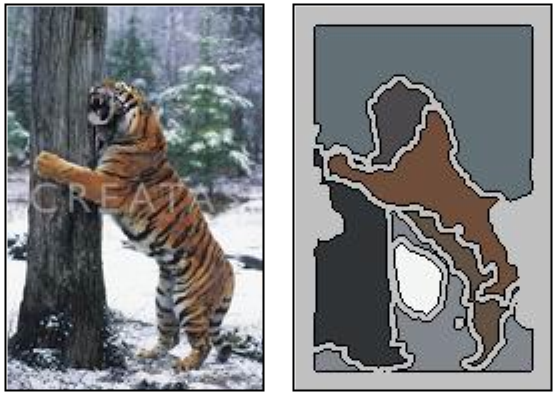
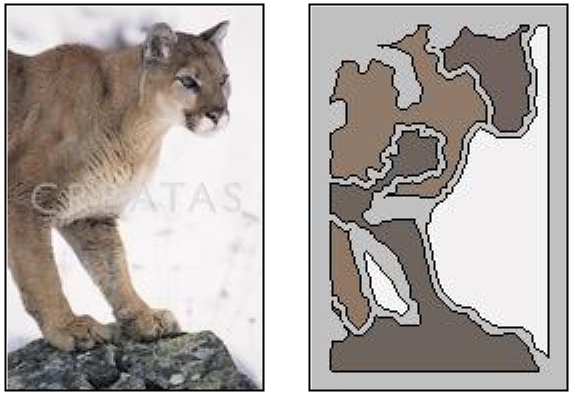
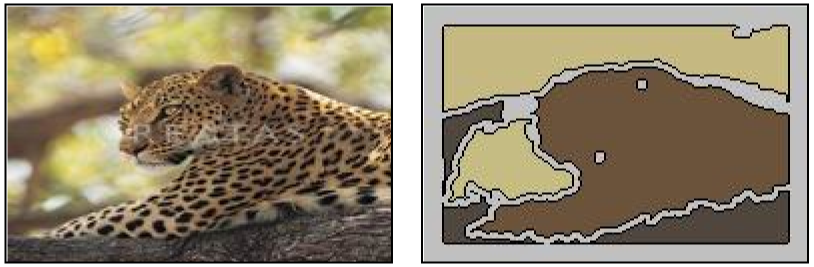
$$p(C_j \mid x_i) = \frac{p(x_i \mid C_j) \cdot p(C_j)}{p(x_i)} = \frac{p(x_i \mid C_j) \cdot p(C_j)}{\sum_j p(x_i \mid C_j) \cdot p(C_j)}$$

  - Maximization Step

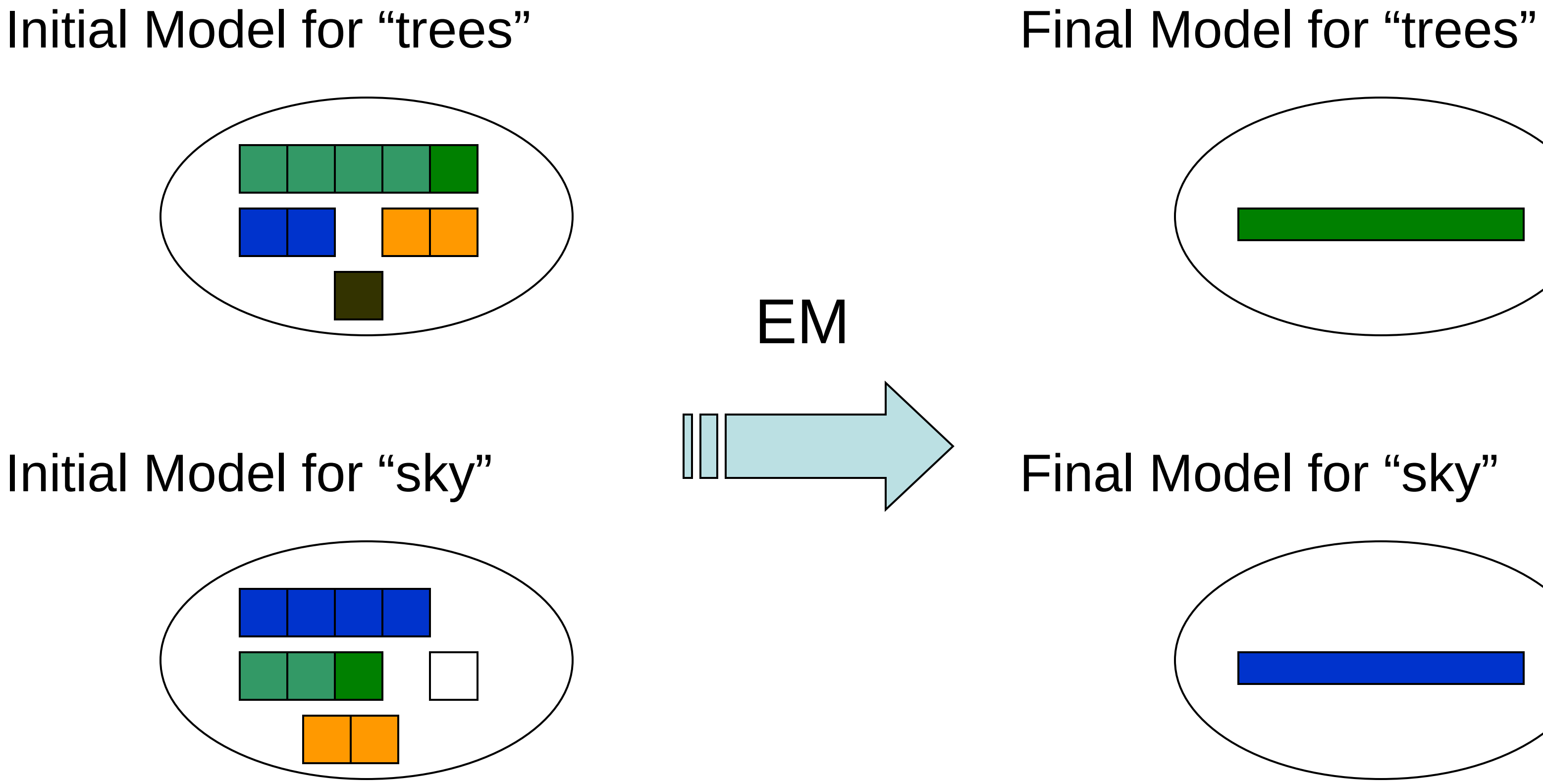
$$\mu_j = \frac{\sum_i p(C_j \mid x_i) \cdot x_i}{\sum_i p(C_j \mid x_i)} \qquad \Sigma_j = \frac{\sum_i p(C_j \mid x_i) \cdot (x_i - \mu_j) \cdot (x_i - \mu_j)^T}{\sum_i p(C_j \mid x_i)} \qquad p(C_j) = \frac{\sum_i p(C_j \mid x_i)}{N}$$

EM Clustering using color and texture information at each pixel

(from Blobworld)

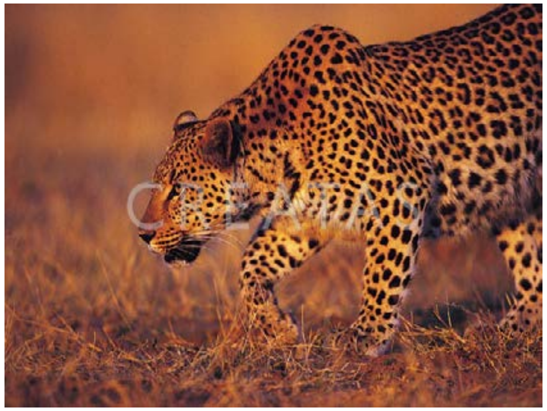





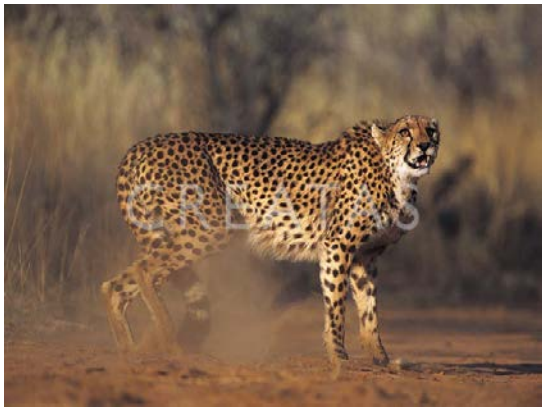
EM for Classification of Images in Terms of their Color Regions

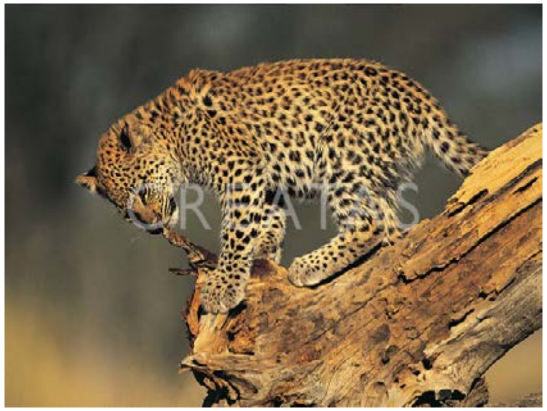


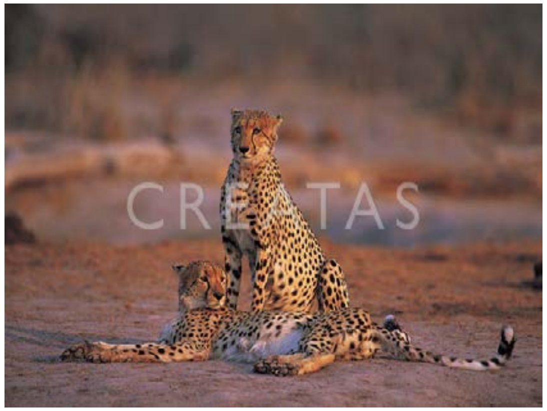
Sample Results

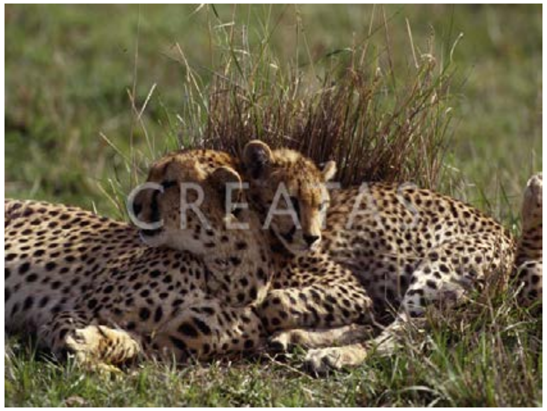
cheetah









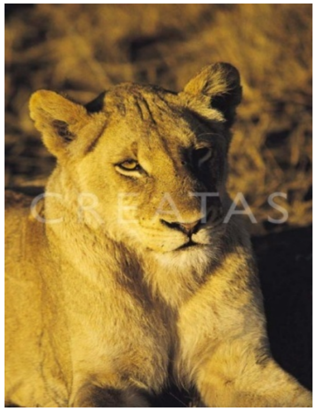


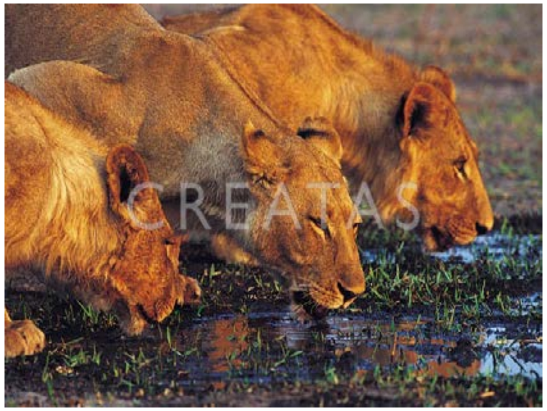
Sample Results (Cont.)

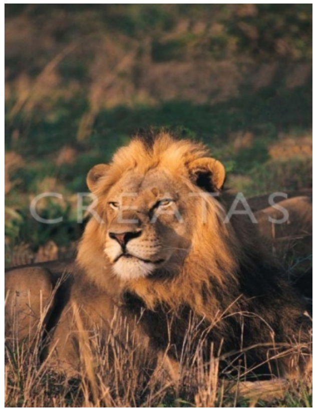
grass

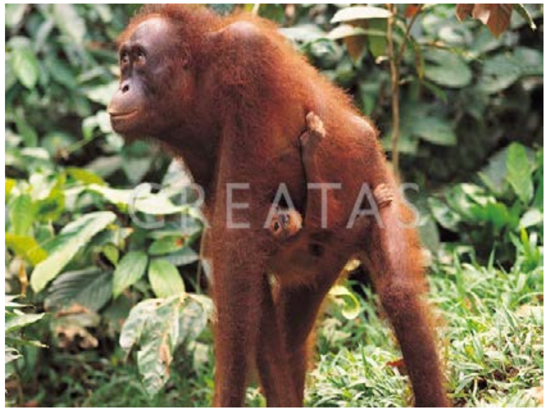
Sample Results (Cont.)

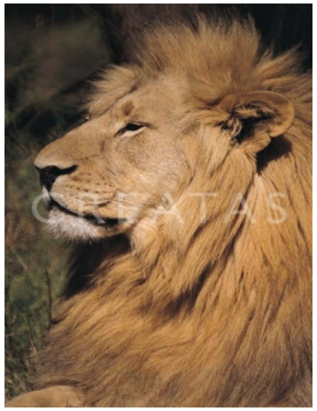
lion







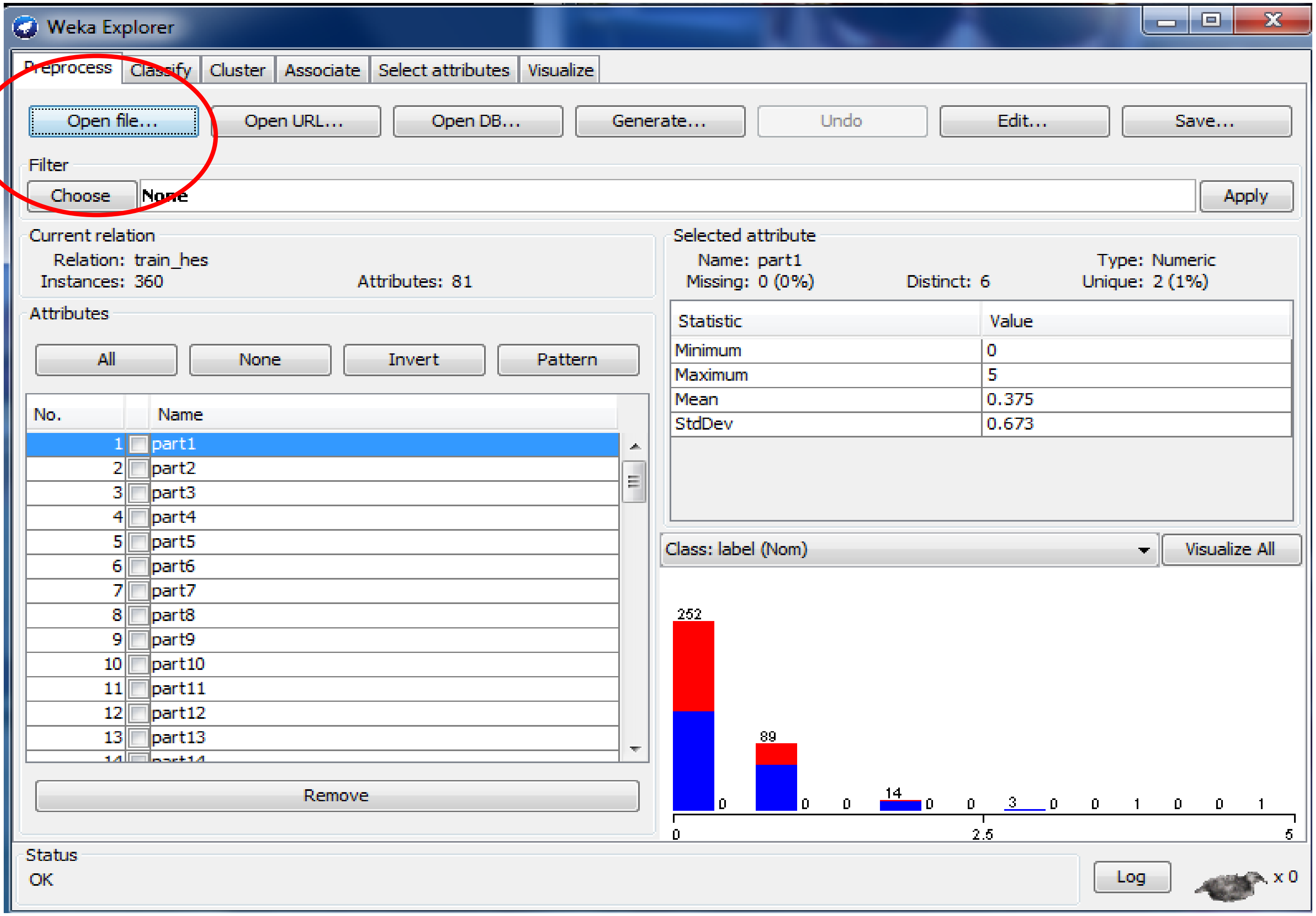




WEKA

- WEKA is a set of data mining tools written in Java from the University of Waikato in New Zealand and is named after a flightless bird.
- WEKA is open source software provided under GNU.
- We use it heavily in our research to test out different classifiers. We may later replace a WEKA classifier with a more efficient C++ version.

Open training set file.



Current relation
Relation: train_hes
Instances: 360
Attributes: 81

No.		Name
1	☐	part1
2	☐	part2
3	☐	part3
4	☐	part4
5	☐	part5
6	☐	part6
7	☐	part7
8	☐	part8
9	☐	part9
10	☐	part10
11	☐	part11
12	☐	part12
13	☐	part13

Selected attribute
Name: part1
Type: Numeric
Missing: 0 (0%)
Distinct: 6
Unique: 2 (1%)

Statistic	Value
Minimum	0
Maximum	5
Mean	0.375
StdDev	0.673

Class: label (Nom)

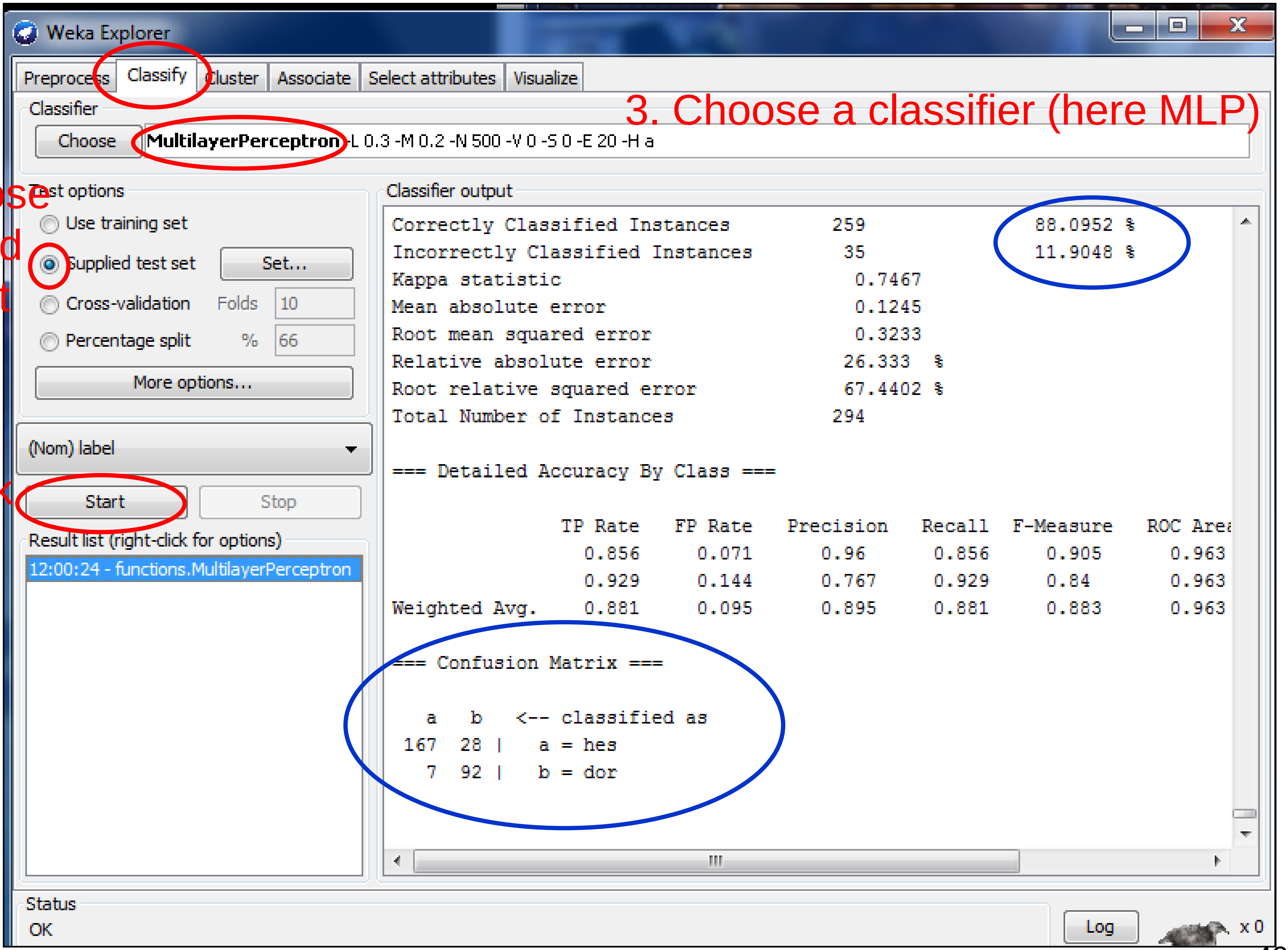
Status
OK

1. Choose classify

2. Choose Supplied Test Set

3. Choose a classifier (here MLP)

4. Click Start and wait.



Classifier: MultilayerPerceptron -L 0.3 -M 0.2 -N 500 -V 0 -S 0 -E 20 -H a

Test options: [✓ Supplied test set]

(Nom) label

Result list: 12:00:24 - functions.MultilayerPerceptron

Status: OK

```
Correctly Classified Instances         259               88.0952 %
Incorrectly Classified Instances        35               11.9048 %
Kappa statistic                          0.7467
Mean absolute error                      0.1245
Root mean squared error                  0.3233
Relative absolute error                 26.333  %
Root relative squared error             67.4402 %
Total Number of Instances              294

=== Detailed Accuracy By Class ===

               TP Rate   FP Rate   Precision   Recall  F-Measure   ROC Area
                 0.856     0.071      0.96      0.856     0.905      0.963
                 0.929     0.144      0.767     0.929     0.84       0.963
Weighted Avg.    0.881     0.095      0.895     0.881     0.883      0.963

=== Confusion Matrix ===

   a   b   <-- classified as
 167  28 |   a = hes
   7  92 |   b = dor
```

Lots of Available Classifiers

Some that we have used.

- Bayes Classifiers
  - Naive Bayes
  - Bayes Nets
- Functions
  - Multilayered Perception
  - SMO (an SVM)
- Metaclassifiers
  - Bagging
  - Adaboost
- Trees
  - REP-tree
  - Random Forest

What Applications Use Machine Learning?

- Computer Vision
- Speech and Natural Language Processing
- Medical Diagnosis
- Predicting Waiting Times in Emergency Rooms
- Financial Planning
- Credit Card Fraud
- Identify Spam