Question 1. (18 points) Suppose we have the following definitions in a Racket program:

```
(define w '((a) b c))
(define x (append (car w) (cdr w)))
(define y (list (cadr w) (cddr w)))
(define z (list (cdr w) y (cddr x)))
```

(a) (12 points) Draw a diagram showing the combined results of evaluating these definitions together in the given order in a newly reset Racket environment.



(b) (6 points) What are the values displayed if w, x, y, and z are printed by Racket?

- w: `((a) b c)
- x: '(a b c)
- y: '(b (c))
- z: `((b c) (b (c)) (c))

Question 2. (18 points) (programming with lists) (a) (9 points) Write a Racket function concat whose argument is a list of lists. The function should return a single list containing all the elements of the original lists. If one of the original lists contains a nested list as an element, that nested list should be included in the result as a single list element. You should not define any additional (auxiliary) functions, local or not, and your solution does not need to be tail-recursive. Hint: you will likely want to use Racket's null? and append functions in your code. Examples:

```
\begin{array}{l} (\text{concat '())} \Rightarrow '() \\ (\text{concat '((a b) (c d))}) \Rightarrow '(a b c d) \\ (\text{concat '(() (1) (2 3) () (4))}) \Rightarrow '(1 2 3 4) \\ (\text{concat '((() w) () (x (y) ((z)))}) \Rightarrow '(() w x (y) ((z))) \end{array}
```

(Sample solution 4 lines – you don't need to match that, but it might give some idea of what's possible.)

```
(define (concat lst) ;; write your code below
  (if (null? lst)
        lst
        (append (car lst) (concat (cdr lst)))))
```

Question 2 (cont.) (b) (9 points) Write a Racket function flatten whose argument is a list of nested lists. The function should return a list with the contents of the original lists, with all inner lists collapsed into a sequence of non-list elements in the original order. As before you should not define any auxiliary functions, but you *will* want to use the function concat from part (a). Examples:

```
(\text{flatten '())} \Rightarrow '()
(\text{flatten '((a b) (c d)))} \Rightarrow '(a b c d)
(\text{flatten '((() w) () (x (y) ((z))))} \Rightarrow '(w x y z)
```

Hint: You will find it helpful to use Racket's list? and map functions in addition to concat. (Sample solution 4 lines.)

```
(define (flatten lst) ;; write your code below
(if (list? lst)
      (concat (map flatten lst))
      (list lst)))
```

Question 3. (16 points, 8 each) For this question you are to give two implementations of a function sum-greater that returns the sum of all numbers in a list that are greater than some threshold. For example, $(sum-greater (2 \ 1 \ 7 \ 5 \ 3 \ 6) \ 4)$ should evaluate to 18 because that is the sum of the three elements of the list (7, 5, and 6) that are greater than 4. You should assume that the list argument contains only numbers and that the threshold (the second argument) is also a number.

(a) Give an implementation of sum-greater using simple recursion and without using any higher-order functions (i.e., no map, filter, foldl, foldr, etc.). For full credit your solution must be tail recursive, and any auxiliary (helper) functions must be defined inside of sum-greater, not externally. (Sample solutions 7-8 lines)

(b) Now give a second implementation of this function that solves the problem using higher-order Racket functions (map, etc.) and, if needed, using lambda to create anonymous functions, but does not have any conditional logic such as if or cond, and does not call any functions you have defined (even recursively). Hints: don't be intimidated by the restrictions – the solution is a fairly straightforward use of higher-order functions and should be much shorter than your version in part (a). Also, recall that foldl combines values in a list using a given operator and identity element: (foldl op id lst) and is left associative. foldr does the same but is right associative. (Sample solution 2 lines)

```
(define (sum-greater lst thresh) ;; write your code below
(foldl + 0 (filter (lambda (elem) (> elem thresh)) lst)))
```

Question 4. (16 points) Environments. Suppose we execute the following definitions in Racket:

(a) (12 points) Draw a diagram showing the bindings, environments, and closures that exist after evaluating the above expressions. You should only include environments that remain bound after the above expressions have finished evaluating, not any other environments that existed during evaluation but are no longer active at the end.



(b) (4 points) What result is printed by Racket if we evaluate the expression (func) after the above definitions have been executed?

`(5 4 2 3)

Question 5. (16 points) Streams. Recall that we can implement a stream in Racket as a thunk (a 0-argument function) that, when called, returns a pair whose car is the current item in the stream and whose cdr is a stream (thunk) that will return the next element of the stream when used appropriately.

Write a Racket function cycle whose argument is a list pattern and that returns a stream whose elements repeatedly cycle through the element of pattern. When the stream reaches the end of pattern it should continue from the beginning and continue cycling indefinitely. Examples:

```
(cycle `(a b c)) => stream representing the sequence 'a 'b 'c 'a 'b 'c ...
(cycle `(a)) => stream representing the sequence 'a 'a 'a 'a 'a ...
```

You can assume that the pattern will contain at least one element. The elements in the pattern might be arbitrarily complex or have any type, but that should not have any effect on your solution.

Hint: you might find it useful to have a private auxiliary function in the stream closure that takes a pattern and returns an updated pattern shifted by one position, e.g., '(a b c d) => '(b c d a) takes. (Sample solutions 7-10 lines)

Question 6. (16 points) Echoes of memo-like things. Suppose we have the following two function definitions, which are identical except that the order of let and lambda are reversed at the beginning of the function definition.

```
(define max-1
  (lambda (arg)
    (let ([max 0])
        (if (< arg max)
            max
            (begin (set! max arg) arg)))))
(define max-2
  (let ([max 0])
        (lambda (arg)
            (if (< arg max)
                max
               (begin (set! max arg) arg)))))</pre>
```

(a) (8 points) Now suppose we evaluate the following expressions *one after the other in the order given below*. Each expression is evaluated after any effects from the previous expressions(s) have occurred. Give the values returned by each expression as it is evaluated in sequence.

 $(max-1 10) \implies 10$ $(max-2 20) \implies 20$ $(max-1 5) \implies 5$ $(max-2 15) \implies 20$

(continued on the next page)

Question 6. (cont.) (b) (8 points) Explain the results you observed in part (a). If changing the order of the let and lambda expressions did not change the results computed by max-1 and max-2 in part (a), explain why not. If it did make a difference, give a brief explanation of what happened. Feel free to include diagrams showing environments and closures if it helps explain things, but this is not required.

The two functions do not have the same behavior. max-1 returns its argument or 0 if the argument is negative, while max-2 returns the maximum value that has ever been passed to it as an argument, or 0 if it has never been called with a non-negative value.

The closure bound to max-1 contains an ordinary (let ...) expression and an environment pointer that refers to the global environment. Each time max-1 is evaluated, a new let binding is created with max initialized to 0, and that environment is discarded after max-1 finishes, even if max was updated by set! during evaluation of max-1.

When the max-2 closure is created, however, the lambda expression is evaluated inside the let environment that initially binds max to 0, and the environment pointer in that closure points to that let environment. That means that every time max-2 is evaluated, the same let environment containing the same max is referenced, max is updated if the new argument is larger than the previous value of max, and the most recent value of max is returned.