Question 1. (14 points) Regular expressions and DFAs. One format for writing dates gives the date first, a 3-letter month abbreviation in the middle, and the year last (for this problem we’ll use 2-digit years to keep things shorter). Examples are 8feb17 (today), 31dec99, 1jan00, and 4jul76. More specifically, a date is a string \texttt{ddmmmyy} or \texttt{dmmmyy}, where:

- \texttt{d} or \texttt{dd} is a date in the range 1-31 with no leading zeros,
- \texttt{mmm} is a month consisting of three lower-case letters
- \texttt{yy} is a year in the range 00 to 99.

To simplify the problem, any date in the range 1-31 is valid for any month (i.e., don’t worry about trying to prevent dates like 30feb12), and any 3-letter sequence of lower-case letters is valid for the month.

You must restrict yourself to the basic regular expression operations covered in class and on homework assignments: \texttt{r s}, \texttt{r | s}, \texttt{r*}, \texttt{r+}, \texttt{r?}, character classes like [a-cxy] and [^aeiou], abbreviations \texttt{name=regexp}, and parenthesized regular expressions. No additional operations that might be found in the “regexp” packages in various Unix programs, scanner generators like JFlex, or language libraries.

(a) (6 points) Give a regular expression that generates all valid dates according to the above rules.

\[
( [12][0-9]? | 3[01]? | [4-9] ) [a-z] [a-z] [a-z] [0-9] [0-9]
\]

(b) (8 points) Draw a DFA that accepts all valid dates according to the above rules. (There is additional space on the next page for your DFA if it doesn’t fit here.)
Question 2. (14 points) Scanners and tokens. Just for fun, we ran our MiniJava scanner using a file containing the following C++ code fragment as input:

```cpp
bool Thing::f(int x) {
    return this->val <= x;
}
```

Below, list in order the tokens that would be returned by a scanner for MiniJava as it reads this input. If there is a lexical error in the input, indicate where that error is encountered by writing a short explanation of the error in between the valid tokens that appear before and after the error(s) (something brief like “illegal character #” if a “#” was found in the file would be fine). The token list should include additional tokens found after any error(s) in the input. You may use any reasonable token names (e.g., LPAREN, ID(x), etc.) as long as your meaning is clear.

A copy of the MiniJava grammar is attached as the last page of the test. You may remove it for reference while you answer this question. You should assume the scanner implements MiniJava syntax as defined in that grammar, with no extensions to the language.

```
ID(bool) ID(Thing)
Invalid character “:”
Invalid character “:”
ID(f) LPAREN INT ID(x) RPAREN LBRACE
RETURN THIS MINUS
Invalid character “>”
ID(val) LESS EQUAL ID(x) SCOLN RBRACE
```
Question 3. (12 points) Ambiguity. Consider the following grammar:

\[
\begin{align*}
P &::= V \mid VX \\
V &::= Vw \mid \epsilon \\
X &::= w
\end{align*}
\]

Is this grammar ambiguous? If so, give a proof that it is by showing two distinct parse trees, or two distinct leftmost (or rightmost) derivations, for some string. If not, give an informal, but precise argument why it is not ambiguous.

Yes.

Here are two distinct leftmost derivations of \( w w \):

\[
\begin{align*}
P &\Rightarrow V \Rightarrow Vw \Rightarrow Vww \Rightarrow w \ w \\
P &\Rightarrow VX \Rightarrow VwX \Rightarrow wX \Rightarrow w \ w
\end{align*}
\]

There are, of course, many other examples.
Question 4. (34 points) The you-can’t-say-you-weren’t-expecting-it parsing question. Here is a tiny grammar.

0. \( S' := S \$ \)  ($$ represents end-of-file)  
1. \( S ::= A \ B \)  
2. \( S ::= c \)  
3. \( A ::= a \)  
4. \( A ::= \varepsilon \)  
5. \( B ::= b \)  
6. \( B ::= B \ b \)

(a) (12 points) Draw the LR(0) state machine for this grammar.

(b) (8 points) Compute nullable and the FIRST and FOLLOW sets for the nonterminals \( S, A, \) and \( B \) in the above grammar:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>nullable</th>
<th>FIRST</th>
<th>FOLLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>no</td>
<td>a, b, c</td>
<td>$</td>
</tr>
<tr>
<td>( A )</td>
<td>yes</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>( B )</td>
<td>no</td>
<td>b</td>
<td>b, $</td>
</tr>
</tbody>
</table>

(continued on next page)
Question 4. (cont.) Grammar repeated from previous page for reference:

0. $S' ::= S \ \$ 
1. $S ::= A \ B 
2. $S ::= c 
3. $A ::= a 
4. $A ::= \epsilon 
5. $B ::= b 
6. $B ::= B \ b 

(c) (10 points) Write the LR(0) parse table for this grammar based on your LR(0) state machine in your answer to part (a).

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>S</th>
<th>S</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td>acc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>r4, s3</td>
<td>r4</td>
<td>r4, s2</td>
<td>r4</td>
<td>g0</td>
<td>g4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>r2</td>
<td>r2</td>
<td>r2</td>
<td>r2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>r3</td>
<td>r3</td>
<td>r3</td>
<td>r3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>s5</td>
<td></td>
<td></td>
<td>g6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>r5</td>
<td>r5</td>
<td>r5</td>
<td>r5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>r1</td>
<td>r1, s7</td>
<td>r1</td>
<td>r1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>r6</td>
<td>r6</td>
<td>r6</td>
<td>r6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(d) (2 points) Is this grammar LR(0)? Why or why not?

No. State 1 has a shift/reduce conflict on inputs a and c, and state 6 has a shift/reduce conflict on input b.

(e) (2 points) Is this grammar SLR? Why or why not?

Yes. In state 1, FOLLOW(A) only contains b, so we would only reduce if b is the next symbol. In state 6, FOLLOW(S) is only the end-of-file marker $, so we would not reduce unless we had reached the end of the input.
Question 5. (16 points) LL parsing. Here is the grammar from the previous question, but without the extra $S'::= S \$ production that we added for the LR parser.

1. $S ::= A \ B$
2. $S ::= c$
3. $A ::= a$
4. $A ::= \epsilon$
5. $B ::= b$
6. $B ::= B \ b$

Does this grammar satisfy the LL(1) condition? Give a technical justification for your answer. If it is not LL(1), change the grammar so that it is suitable for LL(1) parsing without changing the language that it generates.

Hint: It may save some time to compute the FIRST and FOLLOW sets requested in the previous question before working on this one.

This question should have been phrased a bit differently, something like “Can this grammar be used as the basis for a predictive parser with no backtracking (i.e., LL(1)).” The reason is that asking about the LL(1) condition is a bit too narrow. Although it is true that in all of the $X ::= \alpha, X ::= \beta$ productions for the same non-terminal $X$, FIRST($\alpha$) and FIRST($\beta$) are disjoint, that isn’t quite enough when there is a production like $A ::= \epsilon$ in the grammar.

In that case, since $A$ is nullable, we need to look at FOLLOW($A$) when we are deciding between productions 3 and 4 during a top-down parse. Since FOLLOW($A$) is {$b$}, and that is disjoint from FIRST($a$), which is {$a$}, we don’t have a conflict.

We could also fold all of the productions for $A$ into the first production and we get the following rules for $S$:

$$S ::= a \ B \mid B \mid c$$

That eliminates $A$ from the grammar. Since FIRST($B$) is {$b$}, the three possibilities for $S$ have disjoint FIRST sets and that clears up that problem.

Because of the wording of the question, we did not make significant deductions for answers that covered the disjoint FIRST sets but missed the need to check FOLLOW sets or otherwise change the grammar because of the $\epsilon$-production.

The other issue is the direct left recursion in rule 6. This can be eliminated by rewriting productions 5 and 6 as follows, or doing something similar:

5. $B ::= b \ C$
6. $C ::= b \ C \mid \epsilon$
**Question 6.** (10 points, 1 each) Different parts of the front-end of a compiler detect different errors in source programs. For each of the following possible errors, indicate which part of the front-end of a MiniJava compiler would detect the error by filling in the blank with scan, if the scanner would detect the error; parse, if the parser would detect the error; sem, if the semantics/type-checker phase would detect the error; or can’t, if the front end of the compiler cannot or is not guaranteed to detect the error.

*Hint: think carefully about what, exactly, each part of the front end of the compiler does. The MiniJava grammar is attached as the last page of this exam for reference. You may remove it from the exam if you wish.*

**__sem__** Identifier `x` is not declared in the statement `x=17;`

**__sem__** Identifier `x` has type `boolean` in the statement `x=17;`

**__parse__** There is no `<=` operator in MiniJava  
(This one may seem a little surprising, but since `<` and `=` are both valid MiniJava tokens, the scanner will deliver them to the parser, which will then discover that they cannot appear adjacent to each other in a MiniJava program.)

**__scan__** In the expression `i%j`, there is no `%` operator in MiniJava

**__sem__** Boolean values cannot be multiplied by integers (i.e., `4*true` is illegal)

**__parse__** Missing expression following `while` in `while ( ) x=x+1;`

**__scan__** `#` is not a legal character in a MiniJava program

**__can’t__** `x` has the value `-1` in the expression `new int[x]`

**__parse__** Missing object reference in a MiniJava method call (i.e., `f(17)` is not possible but `this.f(17)` is allowed).  
(This is another one that caught several people. The parser will catch this since the grammar can’t generate a method call without “expression.” in front of the method name.)

**__sem__** In the method call `this.f(17)`, the class of the expression `this` does not contain or inherit a method `f` that has a single parameter of type `int`. 