Static Single-Assignment Form

- Overview of SSA IR
  - Constructing SSA graphs
  - SSA-based optimizations
  - Converting back from SSA form

- Source: Appel ch. 19, also an extended discussion in Cooper-Torczon sec. 9.3
Def-Use (DU) Chains

- Common dataflow analysis problem: Find all sites where a variable is used, or find the definition site of a variable used in an expression

- Traditional solution: def-use chains – additional data structure on top of the dataflow graph
  - Link each statement defining a variable to all statements that use it
  - Link each use of a variable to its definition
DU-Chain Drawbacks

- Expensive: if a typical variable has \( N \) uses and \( M \) definitions, the total cost is \( O(N \times M) \)
  - Would be nice if cost were proportional to the size of the program
- Unrelated uses of the same variable are mixed together
  - Complicates analysis
SSA: Static Single Assignment

- IR where each variable has only one definition in the program text
  - This is a single *static* definition, but that definition can be in a loop that is executed dynamically many times
SSA within Basic Blocks

Similar to the *local value numbering* optimization

- **Original**
  
  \[
  \begin{align*}
  a & := b + c \\
  b & := a - d \\
  c & := b + c \\
  d & := a - d \\
  \end{align*}
  \]

- **SSA**
  
  \[
  \begin{align*}
  a_1 & := b_0 + c_0 \\
  b_1 & := a_1 - d_0 \\
  c_1 & := b_1 + c_0 \\
  d_1 & := a_1 - d_0 \\
  \end{align*}
  \]
Merge Points

- The issue is how to handle merge points
- Solution: introduce a $\Phi$-function
  
  $a_3 := \Phi(a_1, a_2)$

- Meaning: $a_3$ is assigned either $a_1$ or $a_2$ depending on which control path is used to reach the $\Phi$-function
Example

Original

\[
\begin{align*}
b &:= M[x] \\
a &:= 0 \\
\text{if } b < 4 \\
a &:= b \\
c &:= a + b
\end{align*}
\]

SSA

\[
\begin{align*}
b_1 &:= M[x0] \\
a_1 &:= 0 \\
\text{if } b_1 < 4 \\
a_2 &:= b_1 \\
a_3 &:= \Phi(a_1, a_2) \\
c_1 &:= a_3 + b_1
\end{align*}
\]
How Does $\Phi$ “Know” What to Pick?

- It doesn’t
  - When we translate the program to executable form, we can add code to copy either value to a common location on each incoming edge
  - For analysis, all we may need to know is the connection of uses to definitions – no need to “execute” anything
Example With Loop

Original

\[
\begin{align*}
    a &:= 0 \\
    b &:= a + 1 \\
    c &:= c + b \\
    a &:= b \times 2 \\
    \text{if } a < N & \Rightarrow \\
    \text{return } c
\end{align*}
\]

SSA

\[
\begin{align*}
    a_1 &:= 0 \\
    a_3 &:= \Phi(a_1, a_2) \\
    b_1 &:= \Phi(b_0, b_2) \\
    c_2 &:= \Phi(c_0, c_1) \\
    b_2 &:= a_3 + 1 \\
    c_1 &:= c_2 + b_2 \\
    a_2 &:= b_2 \times 2 \\
    \text{if } a_2 < N & \Rightarrow \\
    \text{return } c_1
\end{align*}
\]

Notes:
- \(a_0, b_0, c_0\) are initial values of \(a, b, c\) on block entry
- \(b_1\) is dead – can delete later
- \(c\) is live on entry – either input parameter or uninitialized
Converting To SSA Form

- Basic idea
  - First, add $\Phi$-functions
  - Then, rename all definitions and uses of variables by adding subscripts

- Could simply add $\Phi$-functions for every variable at every join point(!)
  - Wastes way too much space and time
  - Not needed
Path-convergence criterion

- Insert a $\Phi$-function for variable $a$ at point $z$ when:
  - There are blocks $x$ and $y$, both containing definitions of $a$, and $x \neq y$
  - There are nonempty paths from $x$ to $z$ and from $y$ to $z$
  - These paths have no common nodes other than $z$
  - $z$ is not in both paths prior to the end (it may appear in one of them)
The start node of the flow graph is considered to define every variable (even if to “undefined”)

Each $\Phi$-function itself defines a variable, so we need to keep adding $\Phi$-functions until things converge
Dominators
One property of SSA is that definitions dominate uses; more specifically:

- If \( x := \Phi(...)x_i,... \) is in block \( n \), then the definition of \( x_i \) dominates the \( i^{th} \) predecessor of \( n \).

- If \( x \) is used in a non-\( \Phi \) statement in block \( n \), then the definition of \( x \) dominates block \( n \).
Dominance Frontiers

- To get a practical algorithm for placing $\Phi$-functions, we need to avoid looking at all combinations of nodes leading from $x$ to $y$

- Instead, use the dominator tree in the flow graph
Dominance Frontiers

- Definitions
  - \( x \) strictly dominates \( y \) if \( x \) dominates \( y \) and \( x \neq y \)
  - The dominance frontier of a node \( x \) is the set of all nodes \( w \) such that \( x \) dominates a predecessor of \( w \), but \( x \) does not strictly dominate \( w \)

- Essentially, the dominance frontier is the border between dominated and undominated nodes
Dominance Frontiers
Dominance Frontier Criterion

- If a node $x$ contains the definition of variable $a$, then every node in the dominance frontier of $x$ needs a $\Phi$-function for $a$.
  - Since the $\Phi$-function itself is a definition, this needs to be iterated until it reaches a fixed-point.

- Theorem: this algorithm places exactly the same set of $\Phi$-functions as the path criterion given previously.
Placing $\Phi$-Functions: Details

- The basic steps are:
  1. Compute the dominance frontiers for each node in the flowgraph
  2. Insert just enough $\Phi$-functions to satisfy the criterion. Use a worklist algorithm to avoid reexamining nodes unnecessarily
  3. Walk the dominator tree and rename the different definitions of variable $a$ to be $a_1, a_2, a_3, ...$
SSA Optimizations

- Given the SSA form, what can we do with it?
- First, what do we know? (i.e., what information is kept in the SSA graph?)
Dead-Code Elimination

- A variable is live iff its list of uses is not empty

- Algorithm to delete dead code:
  
  while there is some variable v with no uses
  if the statement that defines v has no other side effects, then delete it

- Need to remove this statement from the list of uses for its operand variables – which may cause those variables to become dead
Simple Constant Propagation

- If \( c \) is a constant in \( v := c \), any use of \( v \) can be replaced by \( c \)
  - Then update every use of \( v \) to use constant \( c \)
- If the \( c_i \)s in \( v := \Phi(c_1, c_2, ..., c_n) \) are all the same constant \( c \), we can replace this with \( v := c \)
- Can also incorporate copy propagation, constant folding, and others in the same worklist algorithm
Simple Constant Propagation

W := list of all statements in SSA program
while W is not empty
    remove some statement S from W
    if S is \( v := \Phi(c, c, \ldots, c) \), replace S with \( v := c \)
    if S is \( v := c \)
        delete S from the program
        for each statement T that uses v
            substitute c for v in T
    add T to W
Converting Back from SSA

- The meaning of $x := \Phi(x_1, x_2, \ldots, x_n)$ is “set $x := x_1$ if arriving on edge 1, set $x := x_2$ if arriving on edge 2, etc.”

- So, for each $i$, insert $x := x_i$ at the end of predecessor block I

- Issues: lost copies (recall from class; C&T sec 9.3), swap (C&T sec 9.3)
SSA Summary

- Combine information from control flow graph and data flow analysis
- Allows efficient implementation of many optimizations (e.g., constant propagation, dead code elimination, common subexpression elimination)
- Some optimizations are more efficient without SSA (e.g., most loop transformations—fission, fusion, unfolding...)