

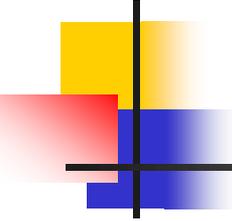
# CSE 401 – Compilers

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Parsing & Context-Free Grammars

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# Agenda for Today

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- Parsing overview
- Context free grammars
- Ambiguous grammars
- Reading: Cooper & Torczon 3.1-3.2
  - Dragon book is also particularly strong on grammars and languages



# Syntactic Analysis / Parsing

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- Goal: Convert token stream to **abstract syntax tree**
- Abstract syntax tree (AST):
  - Captures the structural features of the program
  - Primary data structure for next phases of compilation
- Plan
  - Study how context-free grammars specify syntax
  - Study algorithms for parsing and building ASTs



# Context-free Grammars

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- The syntax of most programming languages can be specified by a context-free grammar (CGF)
- Compromise between
  - REs: can't nest or specify recursive structure
  - General grammars: too powerful, undecidable
- Context-free grammars are a sweet spot
  - Powerful enough to describe nesting, recursion
  - Easy to parse; but also allow restrictions for speed
- Not perfect
  - Cannot capture semantics, as in "variable must be declared" – requires later semantic pass
  - Can be ambiguous



# Derivations and Parse Trees

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- Derivation: a sequence of expansion steps, beginning with a start symbol and leading to a sequence of terminals
- Parsing: inverse of derivation
  - Given a sequence of terminals ( $a_1 a_2 \dots a_n$  tokens) want to recover the nonterminals and structure
- Can represent derivation as a parse tree, that is, a concrete syntax tree



# Example Derivation

*G*

```
program ::= statement | program statement  
statement ::= assignStmt | ifStmt  
assignStmt ::= id = expr ;  
ifStmt ::= if ( expr ) statement  
expr ::= id | int | expr + expr  
Id ::= a | b | c | i | j | k | n | x | y | z  
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

*program*

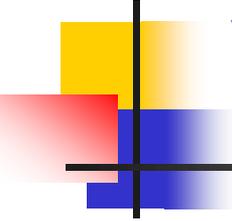
*w* → a = 1 ; if ( a + 1 ) b = 2 ;



# Parsing

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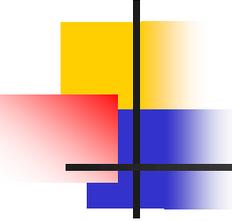
- Parsing: Given a grammar  $G$  and a sentence  $w$  in  $L(G)$ , traverse the derivation (parse tree) for  $w$  in some *standard order* and do *something useful* at each node
  - The tree might not be produced explicitly, but the control flow of the parser corresponds to a traversal



# “Standard Order”

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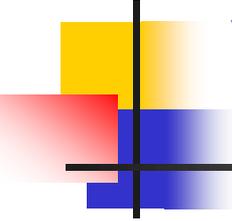
- For practical reasons we want the parser to be *deterministic* (no backtracking), and we want to examine the source program from *left to right*.
  - (i.e., parse the program in linear time in the order it appears in the source file)



# Common Orderings

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- Top-down
  - Start with the root
  - Traverse the parse tree depth-first, left-to-right (leftmost derivation)
  - LL(k), recursive-descent
- Bottom-up
  - Start at leaves and build up to the root
    - Effectively a rightmost derivation in reverse(!)
  - LR(k) and subsets (LALR(k), SLR(k), etc.)



# “Something Useful”

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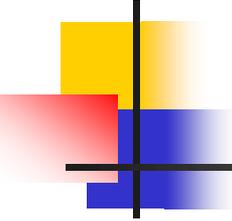
- At each point (node) in the traversal, perform some semantic action
  - Construct nodes of full parse tree (rare)
  - Construct abstract syntax tree (AST) (common)
  - Construct linear, lower-level representation (often produced by traversing initial AST in later phases of production compilers)
  - Generate target code on the fly (used in 1-pass compiler; not common in production compilers)
    - Can't generate great code in one pass, – but useful if you need a quick `n dirty working compiler



# Context-Free Grammars

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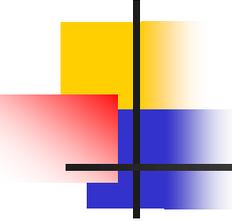
- Formally, a grammar  $G$  is a tuple  $\langle N, \Sigma, P, S \rangle$  where
  - $N$  a finite set of non-terminal symbols
  - $\Sigma$  a finite set of terminal symbols
  - $P$  a finite set of productions
    - A subset of  $N \times (N \cup \Sigma)^*$
  - $S$  the *start symbol*, a distinguished element of  $N$ 
    - If not specified otherwise, this is usually assumed to be the non-terminal on the left of the first production



# Standard Notations

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- $a, b, c$  elements of  $\Sigma$
- $w, x, y, z$  elements of  $\Sigma^*$
- $A, B, C$  elements of  $N$
- $X, Y, Z$  elements of  $N \cup \Sigma$
- $\alpha, \beta, \gamma$  elements of  $(N \cup \Sigma)^*$
- $A \rightarrow \alpha$  or  $A ::= \alpha$  if  $\langle A, \alpha \rangle$  in  $P$



# Derivation Relations (1)

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- $\alpha A \gamma \Rightarrow \alpha \beta \gamma$  iff  $A ::= \beta$  in  $P$ 
  - derives
- $A \Rightarrow^* \alpha$  if there is a chain of productions starting with  $A$  that generates  $\alpha$ 
  - transitive closure



## Derivation Relations (2)

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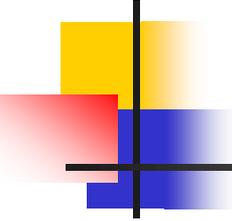
- $w A \gamma \Rightarrow_{lm} w \beta \gamma$  iff  $A ::= \beta$  in  $\mathcal{P}$ 
  - derives **leftmost**
- $\alpha A w \Rightarrow_{rm} \alpha \beta w$  iff  $A ::= \beta$  in  $\mathcal{P}$ 
  - derives **rightmost**
- We will only be interested in leftmost and rightmost derivations – not random orderings



# Languages

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- For  $A$  in  $N$ ,  $L(A) = \{ w \mid A \Rightarrow^* w \}$
- If  $S$  is the start symbol of grammar  $G$ , define  $L(G) = L(S)$ 
  - Nonterminal on left of first rule is taken to be the start symbol if one is not specified explicitly



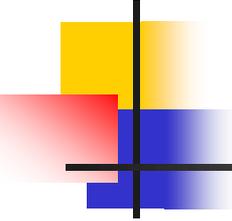
# Reduced Grammars

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- Grammar  $G$  is *reduced* iff for every production  $A ::= \alpha$  in  $G$  there is a derivation

$$S \Rightarrow^* x A z \Rightarrow x \alpha z \Rightarrow^* xyz$$

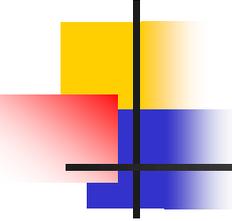
- i.e., no production is useless
- Convention: we will use only reduced grammars



# Ambiguity

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- Grammar  $G$  is *unambiguous* iff every  $w$  in  $L(G)$  has a unique leftmost (or rightmost) derivation
  - Fact: unique leftmost or unique rightmost implies the other
- A grammar without this property is *ambiguous*
  - Note that other grammars that generate the same language may be unambiguous
- We need unambiguous grammars for parsing



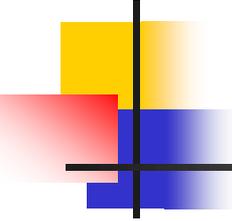
# Example: Ambiguous Grammar for Arithmetic Expressions

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$$\begin{aligned} \text{expr} ::= & \text{expr} + \text{expr} \mid \text{expr} - \text{expr} \\ & \mid \text{expr} * \text{expr} \mid \text{expr} / \text{expr} \mid \text{int} \\ \text{int} ::= & 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

- Exercise: show that this is ambiguous
  - How? Show two different leftmost or rightmost derivations for the same string
  - Equivalently: show two different parse trees for the same string

$expr ::= expr + expr \mid expr - expr$   
 $\mid expr * expr \mid expr / expr \mid int$   
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

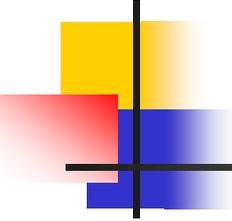


## Example (cont)

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- Give a leftmost derivation of  $2+3*4$  and show the parse tree

$expr ::= expr + expr \mid expr - expr$   
 $\mid expr * expr \mid expr / expr \mid int$   
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

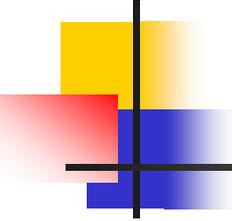


## Example (cont)

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- Give a different leftmost derivation of  $2+3*4$  and show the parse tree

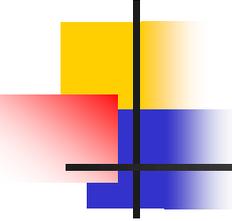
$expr ::= expr + expr \mid expr - expr$   
 $\mid expr * expr \mid expr / expr \mid int$   
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$



## Another example

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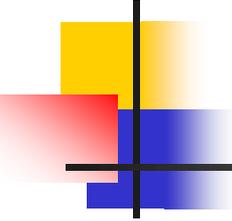
- Give two different derivations of  $5+6+7$



# What's going on here?

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- The grammar has no notion of precedence or associativity
- Traditional solution
  - Create a non-terminal for each level of precedence
  - Isolate the corresponding part of the grammar
  - Force the parser to recognize higher precedence subexpressions first
  - Use left- or right-recursion for left- or right-associative operators (non-associative operators are not recursive)



# Classic Expression Grammar

(first used in ALGOL 60)

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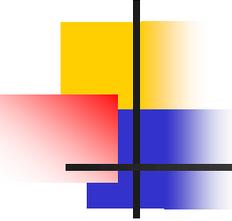
$expr ::= expr + term \mid expr - term \mid term$

$term ::= term * factor \mid term / factor \mid factor$

$factor ::= int \mid ( expr )$

$int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$

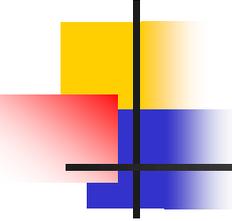
$expr ::= expr + term \mid expr - term \mid term$   
 $term ::= term * factor \mid term / factor \mid factor$   
 $factor ::= int \mid ( expr )$   
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$



Check: Derive  $2 + 3 * 4$

---

$expr ::= expr + term \mid expr - term \mid term$   
 $term ::= term * factor \mid term / factor \mid factor$   
 $factor ::= int \mid ( expr )$   
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$

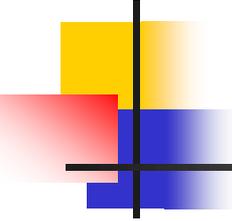


# Check: Derive $5 + 6 + 7$

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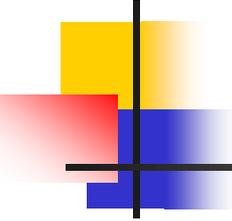
- Note interaction between left- vs right-recursive rules and resulting associativity

$expr ::= expr + term \mid expr - term \mid term$   
 $term ::= term * factor \mid term / factor \mid factor$   
 $factor ::= int \mid ( expr )$   
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$



Check: Derive  $5 + (6 + 7)$

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# Another Classic Example

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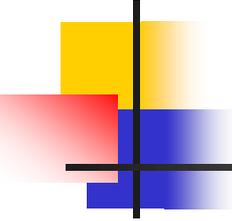
- Grammar for conditional statements

$stmt ::= \text{if } ( cond ) stmt$

$\quad | \text{if } ( cond ) stmt \text{ else } stmt$

- Exercise: show that this is ambiguous
  - How?

$stmt ::= \text{if } ( cond ) stmt$   
 $\quad | \text{if } ( cond ) stmt \text{ else } stmt$

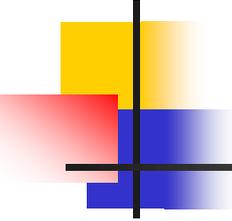


# One Derivation

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$\text{if } ( cond ) \text{ if } ( cond ) stmt \text{ else } stmt$

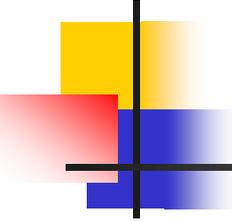
$stmt ::= \text{if } ( cond ) stmt$   
 $\quad | \text{if } ( cond ) stmt \text{ else } stmt$



# Another Derivation

---

$\text{if } ( cond ) \text{ if } ( cond ) stmt \text{ else } stmt$



# Solving “if” Ambiguity

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- Fix the grammar to separate if statements with else clause and if statements with no else
  - Done in Java reference grammar
  - Adds lots of non-terminals
- or, Change the language
  - But it'd better be ok to do this
- or, Use some ad-hoc rule in the parser
  - “else matches closest unpaired if”

# Resolving Ambiguity with Grammar (1)

Stmt ::= MatchedStmt | UnmatchedStmt

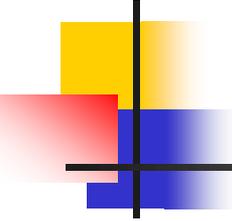
MatchedStmt ::= ... |

**if** ( Expr ) MatchedStmt **else** MatchedStmt

UnmatchedStmt ::= **if** ( Expr ) Stmt |

**if** ( Expr ) MatchedStmt **else** UnmatchedStmt

- formal, no additional rules beyond syntax
- sometimes obscures original grammar



# Check

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```
Stmt ::= MatchedStmt | UnmatchedStmt
MatchedStmt ::= ... |
    if ( Expr ) MatchedStmt else MatchedStmt
UnmatchedStmt ::= if ( Expr ) Stmt |
    if ( Expr ) MatchedStmt else UnmatchedStmt
```

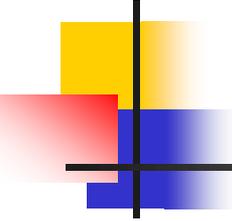
*if* ( *cond* ) *if* ( *cond* ) *stmt* **else** *stmt*

# Resolving Ambiguity with Grammar (2)

- If you can (re-)design the language, avoid the problem entirely

```
Stmt ::= ... |  
       if Expr then Stmt end |  
       if Expr then Stmt else Stmt end
```

- formal, clear, elegant
- allows sequence of Stmts in then and else branches, no { , } needed
- extra end required for every if  
(But maybe this is a good idea anyway?)

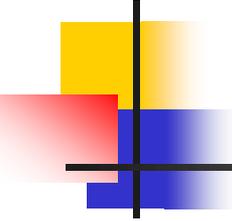


# Parser Tools and Operators

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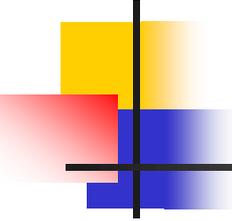
- Most parser tools can cope with ambiguous grammars
  - Makes life simpler if used with discipline
- Typically one can specify operator precedence & associativity
  - Allows simpler, ambiguous grammar with fewer nonterminals as basis for generated parser, without creating problems

# Parser Tools and Ambiguous Grammars



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- Possible rules for resolving other problems
  - Earlier productions in the grammar preferred to later ones
  - Longest match used if there is a choice
- Parser tools normally allow for this
  - But be sure that what the tool does is really what you want



# Coming Attractions

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- Next topic: LR parsing
  - Continue reading ch. 3