LR Parsing
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Agenda
- LR Parsing
- Table-driven Parsers
- Parser States
- Shift-Reduce and Reduce-Reduce conflicts

LR(1) Parsing
- We’ll look at LR(1) parsers
  - Left to right scan, Rightmost derivation, 1 symbol lookahead
  - Almost all practical programming languages have an LR(1) grammar
  - LALR(1), SLR(1), etc. – subsets of LR(1)
    - LALR(1) can parse most real languages, is more compact, and is used by YACC/Bison/CUP/etc.

Bottom-Up Parsing
- Idea: Read the input left to right
- Whenever we’ve matched the right hand side of a production, reduce it to the appropriate non-terminal and add that non-terminal to the parse tree
- The upper edge of this partial parse tree is known as the frontier

Example
- Grammar
  - $S ::= aABe$
  - $A ::= Abc | b$
  - $B ::= d$

- Bottom-up Parse
  - $a b c d e$

Details
- The bottom-up parser reconstructs a reverse rightmost derivation
- Given the rightmost derivation
  $$S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \ldots \Rightarrow \beta_{n-1} \Rightarrow \beta_n \Rightarrow w$$
- the parser will first discover $\beta_{n-1} \Rightarrow \beta_n \Rightarrow w$, then $\beta_{n-2} \Rightarrow \beta_{n-1}$, etc.
- Parsing terminates when
  - $\beta_1$ reduced to $S$ (start symbol, success), or
  - No match can be found (syntax error)
How Do We Parse with This?

- Key: given what we’ve already seen and the next input symbol, decide what to do.
- Choices:
  - Perform a reduction
  - Look ahead further
- Can reduce $A \Rightarrow \beta$ if both of these hold:
  - $A \Rightarrow \beta$ is a valid production
  - $A \Rightarrow \beta$ is a step in this rightmost derivation
- This is known as a *shift-reduce* parser

Sentential Forms

- If $S \Rightarrow^* \alpha$, the string $\alpha$ is called a *sentential form* of the of the grammar
- In the derivation $S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \ldots \Rightarrow \beta_{n-1} \Rightarrow \beta_n = w$ each of the $\beta_i$ are sentential forms
- A sentential form in a rightmost derivation is called a right-sentential form (similarly for leftmost and left-sentential)

Handles

- Informally, a substring of the tree frontier that matches the right side of a production
- Even if $A \Rightarrow \beta$ is a production, $\beta$ is a handle only if it matches the frontier at a point where $A \Rightarrow \beta$ was used in that derivation
- $\beta$ may appear in many other places in the frontier without being a handle for that particular production

Handles (cont.)

- Formally, a *handle* of a right-sentential form $\gamma$ is a production $A \Rightarrow \beta$ and a position in $\gamma$ where $\beta$ may be replaced by $A$ to produce the previous right-sentential form in the rightmost derivation of $\gamma$

Handle Examples

- In the derivation $S \Rightarrow aABe \Rightarrow aAde \Rightarrow aAbcde \Rightarrow abbcde$
- $abbcde$ is a right sentential form whose handle is $A \Rightarrow b$ at position 2
- $aAbcde$ is a right sentential form whose handle is $A \Rightarrow Abc$ at position 4
- Note: some books take the left of the match as the position

Implementing Shift-Reduce Parsers

- Key Data structures
  - A stack holding the frontier of the tree
  - A string with the remaining input
Shift-Reduce Parser Operations

- **Reduce** – if the top of the stack is the right side of a handle $A ::= \beta$, pop the right side $\beta$ and push the left side $A$.
- **Shift** – push the next input symbol onto the stack
- **Accept** – announce success
- **Error** – syntax error discovered

Shift-Reduce Example

<table>
<thead>
<tr>
<th>Stack</th>
<th>Input</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$</td>
<td>abbcde$</td>
<td>shift</td>
</tr>
</tbody>
</table>

Shift-Reduce Example

$$S ::= aA\beta e$$
$$A ::= Abc | b$$
$$B ::= d$$

How Do We Automate This?

- **Def. Viable prefix** – a prefix of a right-sentential form that can appear on the stack of the shift-reduce parser
  - Equivalent: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form
  - Idea: Construct a DFA to recognize viable prefixes given the stack and remaining input
  - Perform reductions when we recognize them

DFA for prefixes of

$S ::= aA\beta e$
$A ::= Abc | b$
$B ::= d$

Trace

<table>
<thead>
<tr>
<th>Stack</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$</td>
<td>abbcde$</td>
</tr>
</tbody>
</table>

Observations

- Way too much backtracking
  - We want the parser to run in time proportional to the length of the input
- Where the heck did this DFA come from anyway?
  - From the underlying grammar
  - We'll defer construction details for now
Avoiding DFA Rescanning

- Observation: after a reduction, the contents of the stack are the same as before except for the new non-terminal on top
  - Scanning the stack will take us through the same transitions as before until the last one
  - If we record state numbers on the stack, we can go directly to the appropriate state when we pop the right hand side of a production from the stack

Stack

- Change the stack to contain pairs of states and symbols from the grammar $s_0 X_1 s_1 X_2 s_2 \ldots X_n s_n$
  - State $s_0$ represents the accept state
    - (Not always added - depends on particular presentation)

  - Observation: in an actual parser, only the state numbers need to be pushed, since they implicitly contain the symbol information, but for explanations, it’s clearer to use both.

Encoding the DFA in a Table

- A shift-reduce parser’s DFA can be encoded in two tables
  - One row for each state
  - action table encodes what to do given the current state and the next input symbol
  - goto table encodes the transitions to take after a reduction

Actions (1)

- Given the current state and input symbol, the main possible actions are
  - $s/ -$ shift the input symbol and state $i$ onto the stack (i.e., shift and move to state $i$)
  - $rj -$ reduce using grammar production $j$
    - The production number tells us how many <symbol, state> pairs to pop off the stack

Actions (2)

- Other possible action table entries
  - accept
  - blank – no transition – syntax error
    - A LR parser will detect an error as soon as possible on a left-to-right scan
    - A real compiler needs to produce an error message, recover, and continue parsing when this happens

Goto

- When a reduction is performed, <symbol, state> pairs are popped from the stack revealing a state uncovered_s on the top of the stack
  - goto[uncovered_s, A] is the new state to push on the stack when reducing production $A ::= \beta$ (after popping $\beta$ and finding state uncovered_s on top)
Reminder: DFA for

\[ S ::= aABe \]
\[ A ::= Abc | b \]
\[ B ::= d \]

LR Parse Table for

<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>s2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>acc</td>
</tr>
<tr>
<td>2</td>
<td>s4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>g3</td>
</tr>
<tr>
<td>3</td>
<td>s6</td>
<td>s5</td>
<td></td>
<td></td>
<td></td>
<td>g8</td>
</tr>
<tr>
<td>4</td>
<td>r3</td>
<td>r3</td>
<td>r3</td>
<td>r3</td>
<td>r3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>r4</td>
<td>r4</td>
<td>r4</td>
<td>r4</td>
<td>r4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>r5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>r7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>r1</td>
<td>r1</td>
<td>r1</td>
<td>r1</td>
<td>r1</td>
<td></td>
</tr>
</tbody>
</table>

LR Parsing Algorithm (1)

```java
word = scanner.getToken();
while (true) {
    s = top of stack;
    if (action[s, word] = si ) {
        push word; push i; // no entry in action table
        report syntax error;
    } else if (action[s, word] = accept ) {
        return;
    } else {
        push word; push i; // no entry in action table
        word = scanner.getToken();
    } else if (action[s, word] = rj ) {
        pop 2 * length of right side of production / |β|;
        uncovered_s = top of stack;
        push left side A of production /; push state goto[uncovered_s, A];
        // no entry in action table
        report syntax error;
        halt or attempt recovery;
    }
} else { // no entry in action table
    report syntax error;
    halt or attempt recovery;
}
```

Example

Stack

<table>
<thead>
<tr>
<th>Stack</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>abcdes</td>
</tr>
</tbody>
</table>

LR States

- Idea is that each state encodes
  - The set of all possible productions that we could be looking at, given the current state of the parse, and
  - Where we are in the right hand side of each of those productions

Items

- An item is a production with a dot in the right hand side
- Example: Items for production \( A ::= XY \)
  \[ A ::= .XY \]
  \[ A ::= X .Y \]
  \[ A ::= XY . \]
- Idea: The dot represents a position in the production
### Problems with Grammars

- Grammars can cause problems when constructing a LR parser
  - Shift-reduce conflicts
  - Reduce-reduce conflicts

### Shift-Reduce Conflicts

- Situation: both a shift and a reduce are possible at a given point in the parse (equivalently: in a particular state of the DFA)
- Classic example: if-else statement
  
  $S ::= \text{ifthen } S \mid \text{ifthen } S \text{ else } S$

### Parser States for

1. $S ::= \text{ifthen } S$
2. $S ::= \text{ifthen } S \text{ else } S$

#### State 3 has a shift-reduce conflict
- Can shift past else into state 4 (s4)
- Can reduce (r1)
  
  $S ::= \text{ifthen } S$

(Note: other $S ::= \text{ifthen}$ items not included in states 2-4 to save space)

### Solving Shift-Reduce Conflicts

- Fix the grammar
  - Done in Java reference grammar, others
- Use a parse tool with a “longest match” rule – i.e., if there is a conflict, choose to shift instead of reduce
  - Does exactly what we want for if-else case
- Guideline: a few shift-reduce conflicts are fine, but be sure they do what you want

### Reduce-Reduce Conflicts

- Situation: two different reductions are possible in a given state
- Contrived example
  
  $S ::= A$
  $S ::= B$
  $A ::= x$
  $B ::= x$
Parser States for

1. $S ::= A$
2. $S ::= B$
3. $A ::= \cdot x$
4. $B ::= \cdot x$

State 2 has a reduce-reduce conflict (r3, r4)

Handling Reduce-Reduce Conflicts

- These normally indicate a serious problem with the grammar.
- Fixes
  - Use a different kind of parser generator that takes lookahead information into account when constructing the states (LR(1) instead of SLR(1) for example)
  - Most practical tools use this information
  - Fix the grammar

Another Reduce-Reduce Conflict

- Suppose the grammar separates arithmetic and boolean expressions
  
  \[
  \text{expr ::= aexp } \mid \text{bexp} \\
  \text{aexp ::= aexp } \cdot \text{aident } \mid \text{aident} \\
  \text{bexp ::= bexp } \&\& \text{bident } \mid \text{bident} \\
  \text{aident ::= id} \\
  \text{bident ::= id}
  \]

- This will create a reduce-reduce conflict

Covering Grammars

- A solution is to merge aident and bident into a single non-terminal (or use id in place of aident and bident everywhere they appear)
- This is a covering grammar
  - Includes some programs that are not generated by the original grammar
  - Use the type checker or other static semantic analysis to weed out illegal programs later

Coming Attractions

- Constructing LR tables
  - We'll present a simple version (SLR(0)) in lecture, then talk about extending it to LR(1)
- LL parsers and recursive descent
- Continue reading ch. 3