



# CSE 401 – Compilers

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LR Parsing  
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## Agenda

- LR Parsing
- Table-driven Parsers
- Parser States
- Shift-Reduce and Reduce-Reduce conflicts

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## LR(1) Parsing

- We'll look at LR(1) parsers
  - Left to right scan, Rightmost derivation, 1 symbol lookahead
  - Almost all practical programming languages have an LR(1) grammar
  - LALR(1), SLR(1), etc. – subsets of LR(1)
    - LALR(1) can parse most real languages, is more compact, and is used by YACC/Bison/CUP/etc.

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## Bottom-Up Parsing

- Idea: Read the input left to right
- Whenever we've matched the right hand side of a production, reduce it to the appropriate non-terminal and add that non-terminal to the parse tree
- The upper edge of this partial parse tree is known as the *frontier*

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## Example

- Grammar
- Bottom-up Parse

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

a b b c d e

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## Details

- The bottom-up parser reconstructs a reverse rightmost derivation
- Given the rightmost derivation  $S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = w$  the parser will first discover  $\beta_{n-1} \Rightarrow \beta_n$ , then  $\beta_{n-2} \Rightarrow \beta_{n-1}$ , etc.
- Parsing terminates when
  - $\beta_1$  reduced to  $S$  (start symbol, success), or
  - No match can be found (syntax error)

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## How Do We Parse with This?

- Key: given what we've already seen and the next input symbol, decide what to do.
- Choices:
  - Perform a reduction
  - Look ahead further
- Can reduce  $A \Rightarrow \beta$  if both of these hold:
  - $A \Rightarrow \beta$  is a valid production
  - $A \Rightarrow \beta$  is a step in *this* rightmost derivation
- This is known as a *shift-reduce* parser

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## Sentential Forms

- If  $S \Rightarrow^* \alpha$ , the string  $\alpha$  is called a *sentential form* of the grammar
- In the derivation  $S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = w$  each of the  $\beta_j$  are sentential forms
- A sentential form in a rightmost derivation is called a *right-sentential form* (similarly for leftmost and left-sentential)

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## Handles

- Informally, a substring of the tree frontier that matches the right side of a production
  - Even if  $A ::= \beta$  is a production,  $\beta$  is a handle only if it matches the frontier at a point where  $A ::= \beta$  was used in that derivation
  - $\beta$  may appear in many other places in the frontier without being a handle for that particular production

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## Handles (cont.)

- Formally, a *handle* of a right-sentential form  $\gamma$  is a production  $A ::= \beta$  and a position in  $\gamma$  where  $\beta$  may be replaced by  $A$  to produce the previous right-sentential form in the rightmost derivation of  $\gamma$

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## Handle Examples

- In the derivation  $S \Rightarrow aAbc \Rightarrow aAde \Rightarrow aAbcde \Rightarrow abcde$ 
  - $abcde$  is a right sentential form whose handle is  $A ::= b$  at position 2
  - $aAbcde$  is a right sentential form whose handle is  $A ::= abc$  at position 4
    - Note: some books take the left of the match as the position

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## Implementing Shift-Reduce Parsers

- Key Data structures
  - A stack holding the frontier of the tree
  - A string with the remaining input

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## Shift-Reduce Parser Operations

- *Reduce* – if the top of the stack is the right side of a handle  $A ::= \beta$ , pop the right side  $\beta$  and push the left side  $A$ .
- *Shift* – push the next input symbol onto the stack
- *Accept* – announce success
- *Error* – syntax error discovered

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## Shift-Reduce Example

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input	Action
\$	abcde\$	<i>shift</i>

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## How Do We Automate This?

- Def. *Viable prefix* – a prefix of a right-sentential form that can appear on the stack of the shift-reduce parser
  - Equivalent: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form
- Idea: Construct a DFA to recognize viable prefixes given the stack and remaining input
  - Perform reductions when we recognize them

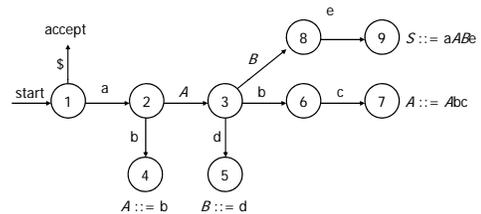
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## DFA for prefixes of

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$



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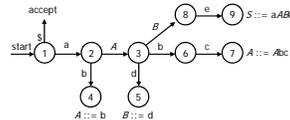
D-16

## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack  
\$

Input  
abcde\$



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## Observations

- Way too much backtracking
  - We want the parser to run in time proportional to the length of the input
- Where the heck did this DFA come from anyway?
  - From the underlying grammar
  - We'll defer construction details for now

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## Avoiding DFA Rescanning

- Observation: after a reduction, the contents of the stack are the same as before except for the new non-terminal on top
  - ∴ Scanning the stack will take us through the same transitions as before until the last one
  - ∴ If we record state numbers on the stack, we can go directly to the appropriate state when we pop the right hand side of a production from the stack

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## Stack

- Change the stack to contain pairs of states and symbols from the grammar
$$s_0 X_1 s_1 X_2 s_2 \dots X_n s_n$$
  - State  $s_0$  represents the accept state
    - (Not always added – depends on particular presentation)
- Observation: in an actual parser, only the state numbers need to be pushed, since they implicitly contain the symbol information, but for explanations, it's clearer to use both.

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## Encoding the DFA in a Table

- A shift-reduce parser's DFA can be encoded in two tables
  - One row for each state
  - *action* table encodes what to do given the current state and the next input symbol
  - *goto* table encodes the transitions to take after a reduction

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## Actions (1)

- Given the current state and input symbol, the main possible actions are
  - *si* – shift the input symbol and state *i* onto the stack (i.e., shift and move to state *i*)
  - *rj* – reduce using grammar production *j*
    - The production number tells us how many <symbol, state> pairs to pop off the stack

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## Actions (2)

- Other possible *action* table entries
  - *accept*
  - blank – no transition – syntax error
    - A LR parser will detect an error as soon as possible on a left-to-right scan
    - A real compiler needs to produce an error message, recover, and continue parsing when this happens

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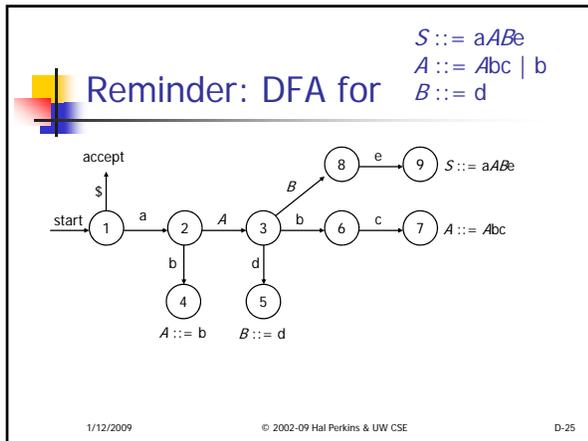
## Goto

- When a reduction is performed, <symbol, state> pairs are popped from the stack revealing a state *uncovered\_s* on the top of the stack
- *goto*[*uncovered\_s*, *A*] is the new state to push on the stack when reducing production  $A ::= \beta$  (after popping  $\beta$  and finding state *uncovered\_s* on top)

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### LR Parse Table for

- $S ::= aABe$
- $A ::= Abc$
- $A ::= b$
- $B ::= d$

State	action						goto		
	a	b	c	d	e	\$	A	B	S
1	s2					acc			g1
2		s4					g3		
3		s6		s5				g8	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

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### LR Parsing Algorithm (1)

```

word = scanner.getToken();
while (true) {
    s = top of stack;
    if (action[s, word] = s') {
        push word; push / (state);
        word = scanner.getToken();
    } else if (action[s, word] = r') {
        pop 2 * length of right side of
        production j (2*|β|);
        uncovered_s = top of stack;
        push left side A of production j;
        push state goto[uncovered_s, A];
    } else if (action[s, word] = accept) {
        return;
    } else {
        // no entry in action table
        report syntax error;
        halt or attempt recovery;
    }
}

```

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### Example

- $S ::= aABe$
- $A ::= Abc$
- $A ::= b$
- $B ::= d$

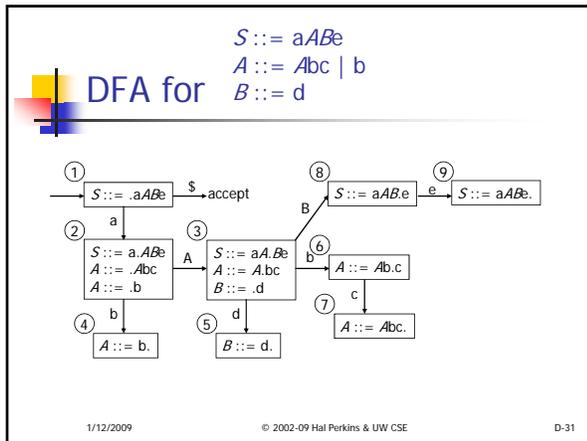
Stack: \$      Input: abbcde\$

S	action						goto		
	a	b	c	d	e	\$	A	B	S
1	s2					ac			g1
2		s4					g3		
3		s6		s5				g8	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

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- ### LR States
- Idea is that each state encodes
    - The set of all possible productions that we could be looking at, given the current state of the parse, and
    - Where we are in the right hand side of each of those productions
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- ### Items
- An *item* is a production with a dot in the right hand side
  - Example: Items for production  $A ::= XY$ 
    - $A ::= .XY$
    - $A ::= X.Y$
    - $A ::= XY.$
  - Idea: The dot represents a position in the production
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### Problems with Grammars

- Grammars can cause problems when constructing a LR parser
  - Shift-reduce conflicts
  - Reduce-reduce conflicts

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### Shift-Reduce Conflicts

- Situation: both a shift and a reduce are possible at a given point in the parse (equivalently: in a particular state of the DFA)
- Classic example: if-else statement
 
$$S ::= \text{ifthen } S \mid \text{ifthen } S \text{ else } S$$

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### Parser States for

- $S ::= \text{ifthen } S$
- $S ::= \text{ifthen } S \text{ else } S$

- State 3 has a shift-reduce conflict
  - Can shift past else into state 4 (s4)
  - Can reduce (r1)  $S ::= \text{ifthen } S$

(Note: other  $S ::= \text{ifthen } S$  items not included in states 2-4 to save space)

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### Solving Shift-Reduce Conflicts

- Fix the grammar
  - Done in Java reference grammar, others
- Use a parse tool with a "longest match" rule – i.e., if there is a conflict, choose to shift instead of reduce
  - Does exactly what we want for if-else case
  - Guideline: a few shift-reduce conflicts are fine, but be sure they do what you want

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### Reduce-Reduce Conflicts

- Situation: two different reductions are possible in a given state
- Contrived example
 
$$S ::= A$$

$$S ::= B$$

$$A ::= x$$

$$B ::= x$$

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## Parser States for

1.  $S ::= A$
2.  $S ::= B$
3.  $A ::= x$
4.  $B ::= x$

①

$S ::= \cdot A$   
 $S ::= \cdot B$   
 $A ::= \cdot x$   
 $B ::= \cdot x$

↓ x

②

$A ::= x \cdot$   
 $B ::= x \cdot$

- State 2 has a reduce-reduce conflict (r3, r4)

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## Handling Reduce-Reduce Conflicts

- These normally indicate a serious problem with the grammar.
- Fixes
  - Use a different kind of parser generator that takes lookahead information into account when constructing the states (LR(1) instead of SLR(1) for example)
    - Most practical tools use this information
  - Fix the grammar

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## Another Reduce-Reduce Conflict

- Suppose the grammar separates arithmetic and boolean expressions
 

```

      expr ::= aexp | bexp
      aexp ::= aexp * aident | aident
      bexp ::= bexp && bident | bident
      aident ::= id
      bident ::= id
      
```
- This will create a reduce-reduce conflict

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## Covering Grammars

- A solution is to merge *aident* and *bident* into a single non-terminal (or use *id* in place of *aident* and *bident* everywhere they appear)
- This is a *covering grammar*
  - Includes some programs that are not generated by the original grammar
  - Use the type checker or other static semantic analysis to weed out illegal programs later

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## Coming Attractions

- Constructing LR tables
  - We'll present a simple version (SLR(0)) in lecture, then talk about extending it to LR(1)
- LL parsers and recursive descent
- Continue reading ch. 3

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