CSE 373: Graphs
(Traversals, Shortest Paths)

Chapter 9

Depth-First Search (DFS)

A fundamental method for traversing a graph:
- start from a particular node...
- walk along a single path visiting nodes
- go back and try a different path only when stuck
**DFS on Directed Graphs**

Also applicable to directed graphs...

- Similar to pre-order traversal on tree/DFS on maze
- Running time?
- edges walked by DFS form a *spanning tree*
- how would we implement this?

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**DFS Implementation**

```
void DFS(vertex v) {

}
```
Breadth-First Search (BFS)

Another way of traversing a graph
- start from a particular node...
- take one step along each of its edges
- then take one step per edge for each of those nodes

BFS on Directed Graphs

Also applicable to directed graphs...
- Running time?
- visited edges again form a spanning tree
- how would we implement this?
BFS Implementation

void BFS(vertex v) {

}

Topological Sort

A notion of ordering the vertices in a DAG:
- If there is a path from \( u \) to \( v \), \( u \) must appear before \( v \) in the ordering
- OR, a vertex \( v \) may not be printed out until all vertices with edges leading to it have been printed
Topological Sort on non-DAGs

Why don’t topological sorts make sense on graphs that aren’t DAGs?

Topological Sort Applications

- Given a graph representing course prerequisites, topological sorts would indicate legal course schedules

- Given a callgraph for a non-recursive C program, topological sorts would indicate function orderings for the source file such that no prototypes are needed
Using Searches for Topological Sort?

- Would a depth-first search visit nodes in topological order?

- Would a breadth-first search?

- What would?

Naive Topological Sort

- Naive algorithm:
  - Keep track of each vertex’s in-degree: the number of edges leading into it
  - Scan the vertex list looking for one whose in-degree is zero
  - Print that vertex out
  - Decrement the in-degrees of all adjacent vertices

- Running Time?
- How could this be improved?
**Improved Topological Sort**

```java
void TopSort(Graph G) {
  // Algorithm details...
}

Running Time?
```

**Shortest Path Problems**

Given a graph $G = (V,E)$ and a vertex $s \in V$, find the shortest path from $s$ to all other vertices.

Many variations:
- unweighted graphs
- weighted graphs with no negative weights
- weighted graphs with negative weights
- weighted acyclic graphs
Unweighted Shortest Path Problem

Assume source vertex is C...

*distance to:*  A  B  C  D  E  F  G  H

What approach could we use to implement this?

The Problem With Negative Weights

*case 1:*

*case 2:*