CSE 373: Asymptotic Analysis

Chapter 2

Recall: FindMajor()

/* print students in a course with major */
void FindMajor(course c, dept major) {
    int i;

    for (i=0; i<num_students; i++) {
        if (c[i].major == major) {
            cout << c[i].first << c[i].last;
        }
    }
}

How fast is this routine?
Exact Times are Tricky

How much time does \texttt{FindMajor()} require?
- number of iterations \times work per iteration:

Simplifying Assumption

Constants are insignificant compared to the \textit{asymptotic} behavior of the program
- expressed as a function of the problem size
- expressed using functions like: \( n, n^2, \log n, 2^n \), etc.
Getting some Intuition...

Using the Computer...
On A Larger Scale...

Ignoring $2^n$
Ignoring $n^3$

On Yet a Larger Scale
Smallest Functions Only

The Moral

Performance can be broken down into primary and secondary effects:

- primary effects: asymptotic growth pattern
- secondary effects: constant factors, less significant terms

- In this class, we'll mainly be concerned with primary effects (asymptotic analysis)
- In the real world, secondary effects are also often worth paying attention to (after the primary ones)
Formally...

Given an algorithm whose running time is $T(n)$...

- $T(n) = O(f(n))$ if there are positive constants $c$ and $n_0$ such that $T(n) \leq c \cdot f(n)$ for all $n \geq n_0$
  - $\log n$, $n$, $100n = O(n)$
- $T(n) = \Omega(f(n))$ if there are positive constants $c$ and $n_0$ such that $T(n) \geq c \cdot f(n)$ for all $n \geq n_0$
  - $n$, $n^2$, $100 \cdot 2^n$, $n \log n = \Omega(n)$
- $T(n) = \Theta(f(n))$ if $T(n) = O(f(n))$ and $T(n) = \Omega(f(n))$
  - $n$, $2n$, $100n$, $n + \log n = \Theta(n)$

Typical Growth Rates (in order)

- $\text{constant: } O(1)$
- $\text{logarithmic: } O(\log n)$
- $\text{log-squared: } O(\log^2 n)$
- $\text{linear: } O(n)$
- $\text{“n log n”: } O(n \log n)$
- $\text{quadratic: } O(n^2)$
- $\text{cubic: } O(n^3)$
- $\text{exponential: } O(2^n)$
General Rules of Thumb

- Constant factors can always be dropped
  - \(5n = O(n)\)
- In sums, smaller terms can always be dropped
  - \(3n \cdot \log n + n^2 + \log n = O(n^2)\)

Analyzing Code

- \textit{C/C++ ops} – constant value
- \textit{consecutive statements} – add individual costs
- \textit{loops} – multiply cost of loop body by number of iterations
- \textit{conditionals} – maximum cost of branches
- \textit{function calls} – evaluate cost of function body

\textit{Above all, use your brain}
Reconsider: FindMajor()

```c
/* print students in a course with major */
void FindMajor(course c, dept major) {
    int i;
    for (i=0; i<num_students; i++) {
        if (c[i].major == major) {
            cout << c[i].first << c[i].last;
        }
    }
}
```

How fast is this routine? At best?
At worst?
On average?

FindAMajor()

```c
/* return pointer to a student in major */
student *FindAMajor(course c, dept major) {
    int i;
    for (i=0; i<num_students; i++) {
        if (c[i].major == major) {
            return &(c[i]);
        }
    }
}
```

What’s the best case for this routine?
The worst case?
The average case?
Asymptotic Analysis

- Determine what characterizes a problem’s size
- Express how much time and memory an algorithm requires as a function of its problem size using $O()$, $\Omega()$, or $\Theta()$
  - worst case
  - best case
  - average case
  - common case
  - overall

Examples from Lecture

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<th>Prob Size</th>
<th>Space</th>
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<th>Worst Time</th>
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