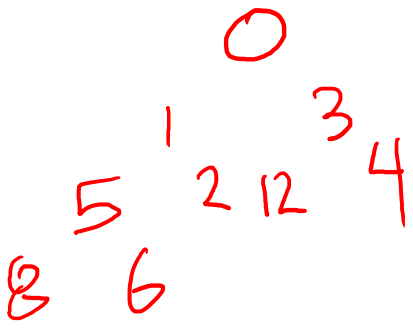
Sorting

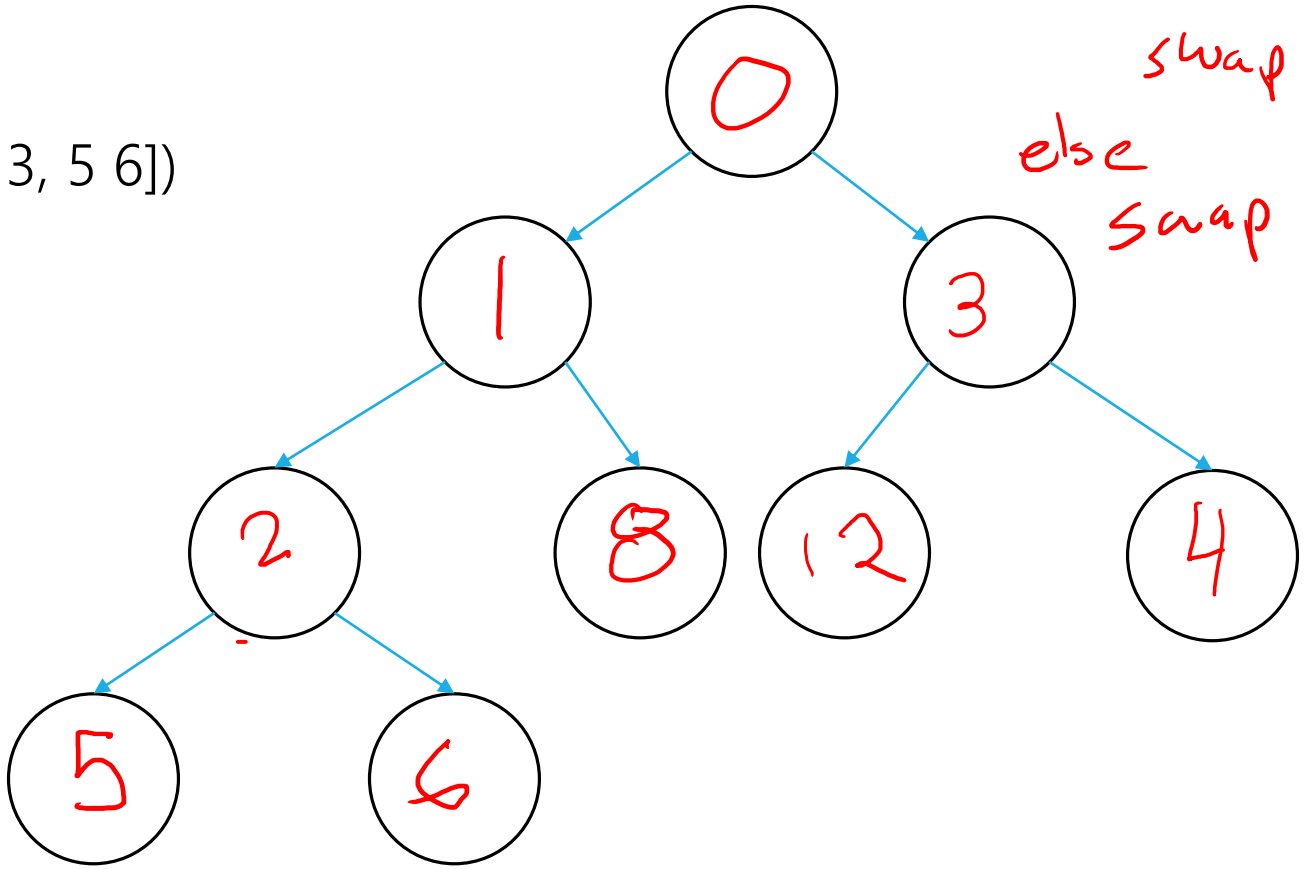
Data Structures and Algorithms

Warmup

buildHeap([8, 2, 4, 0, 1, 12, 3, 5 6])

if (left child < right child)
swap left
else
swap right





0	1	3	2	8	12	4	5	6

Sorting

Take this:

3, 7, 0, 6, 9, 8, 1, 2, 5, 8

And make this:

0, 1, 2, 3, 5, 6, 7, 8, 8, 9

Or this:

9, 8, 8, 7, 6, 5, 3, 2, 1, 0

Comparison Sorts

Compare two elements at a time

General sort, works for most types of elements

Element must form a "consistent, total ordering"

For every element a, b and c in the list the following must be true:

- If a <= b and b <= a then a = b
- If a <= b and b <= c then a <= c
- Either a <= b is true or <= a

What does this mean? **compareTo()** works for your elements

Comparison sorts run at fastest **O(nlog(n))** time

Niche Sorts aka "linear sorts"

Leverages specific properties about the items in the list to achieve faster runtimes

niche sorts typically run O(n) time

In this class we'll focus on comparison sorts

Why Not Just Heap Sort? It's O(n log n)…

In Place sort

A sorting algorithm is in-place if it requires only O(1) extra space to sort the array

Typically modifies the input collection

Useful to minimize memory usage

Stable sort

A sorting algorithm is stable if any equal items remain in the same relative order before and after the sort

Why do we care?

- Sometimes we want to sort based on some, but not all attributes of an item
- Items that "compareTo()" the same might not be exact duplicates
- Enables us to sort on one attribute first then another etc…

[(8, "fox"), (9, "dog"), (4, "wolf"), (8, "cow")]

[(4, "wolf"), (8, "fox"), (8, "cow"), (9, "dog")] Stable

[(4, "wolf"), (8, "cow"), (8, "fox"), (9, "dog")] Unstable

Other Considerations

Worst Case, Average Case

External Sorts (can't fit in memory)

Good for small data sets

Ease of implementation

LinkedList vs Array

https://www.youtube.com/watch?v=Ns4TPTC8whw

Selection Sort

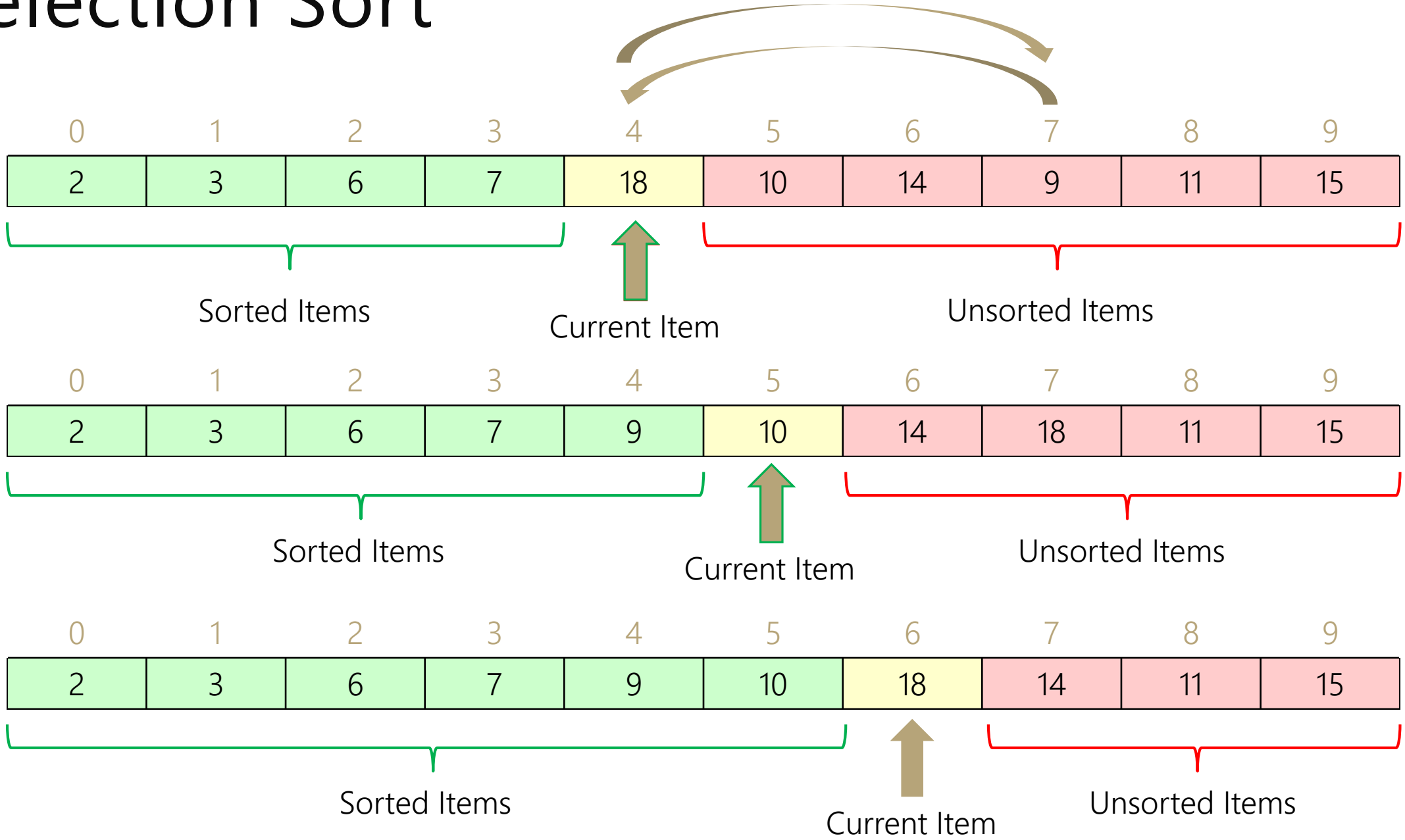
Basic Idea:

Repeatedly scan through the list, moving the next smallest element to the front.

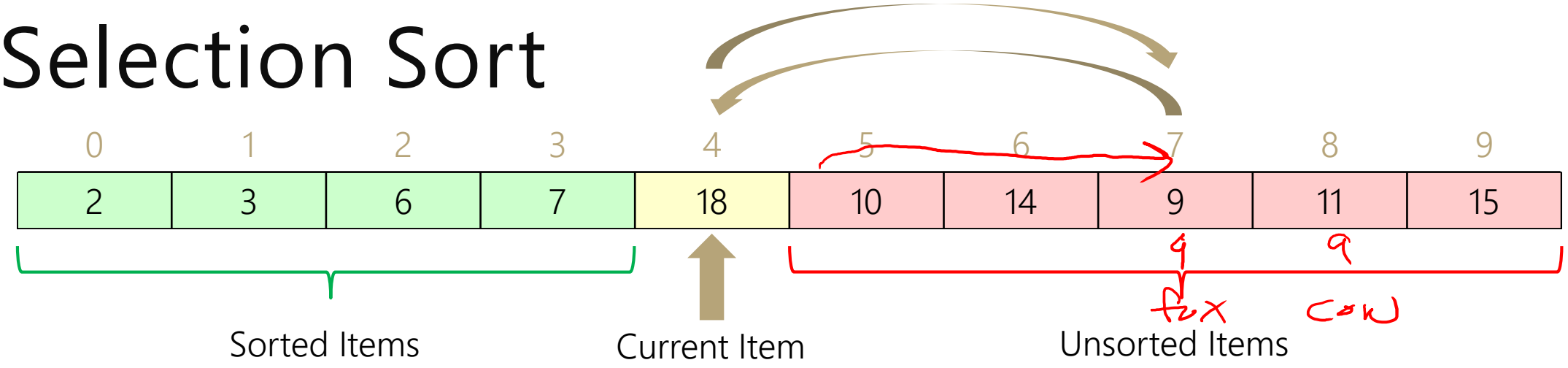
This sounds a lot like heap sort, but worse…

https://www.youtube.com/watch?v=Ns4TPTC8whw

Selection Sort



Selection Sort




```
public void selectionSort(collection) {
   for (entire list)
      int newIndex = findNextMin(currentItem);
      swap(newIndex, currentItem);
}
public int findNextMin(currentItem) {
   min = currentItem
   for (unsorted list)
      if (item < min)
         min = currentItem
   return min
}
public int swap(newIndex, currentItem) {
   temp = currentItem
   currentItem = newIndex
   newIndex = currentItem
}
```

Worst case runtime?	$O(n^2)$
Best case runtime?	$O(n^2)$
Average runtime?	$O(n^2)$
Stable?	Yes
In-place?	Yes

Insertion Sort

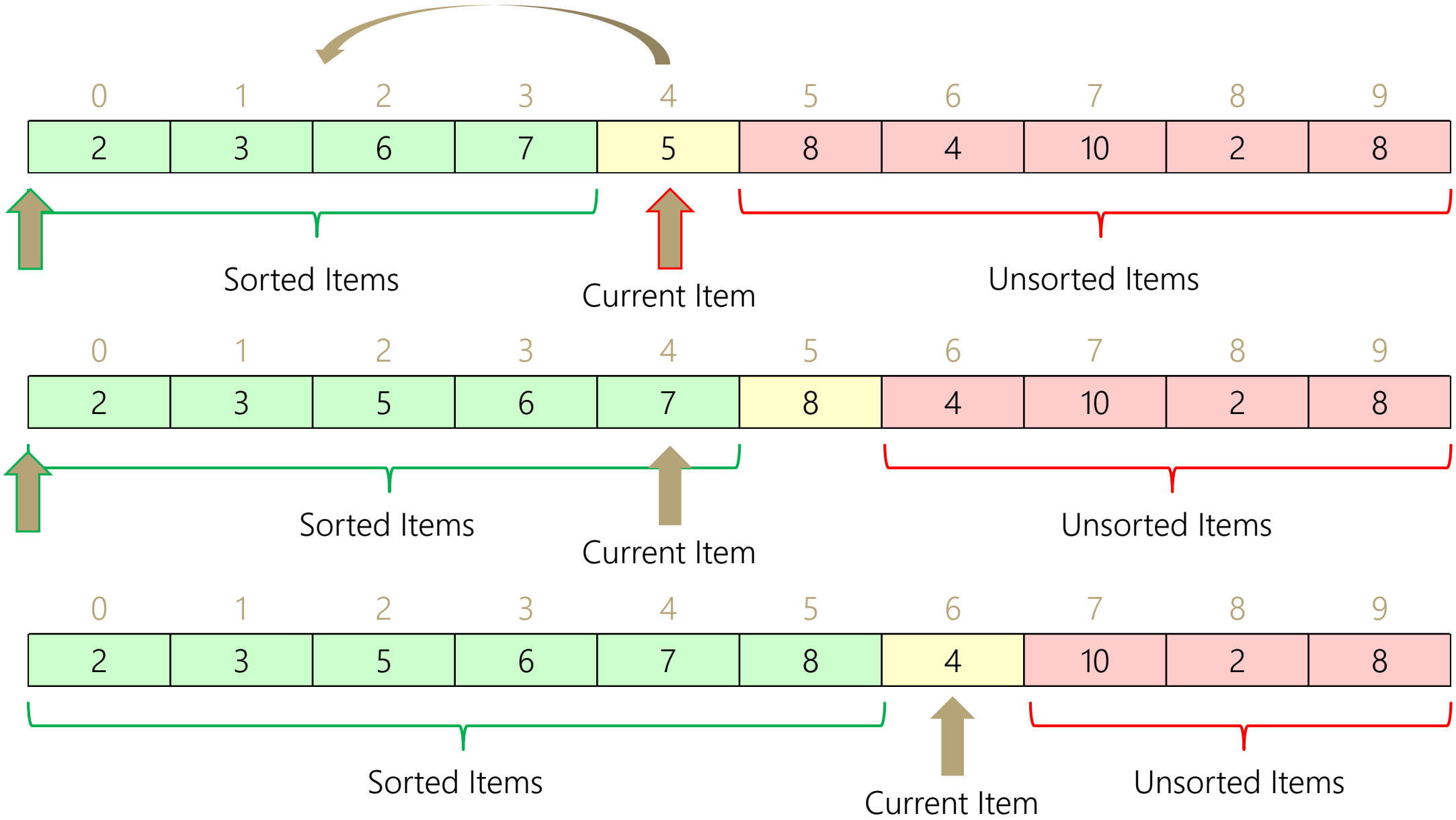
https://www.youtube.com/watch?v=ROalU379l3U

Basic Idea:

Like you would sort a hand of cards – pull out the next card, then insert it into it where it belongs

Insertion Sort

https://www.youtube.com/watch?v=ROalU379l3U



Insertion Sort

for i = 1 to n
for j = 0 to i
do stuff

$\sum_{i=1}^{n} \sum_{j=0}^{i}$

$\underbrace{1+2+3+4+5+\dots+n}$

$\frac{(n+1)n}{2} = O(n^2)$

0	1	2	3	4	5	6	7	8	9
2	3	5	6	7	8	4	10	2	8

Sorted Items

Current Item

Unsorted Items

1 2 3 4 5 6 7 8 | 6_2

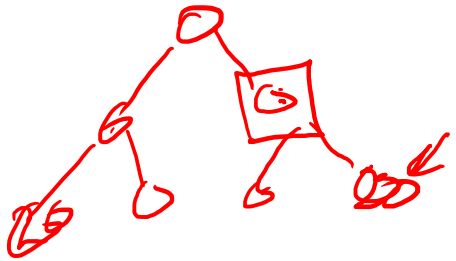
```
public void insertionSort(collection) {
   for (entire list)
      if(currentItem is bigger than nextItem)
         int newIndex = findSpot(currentItem);
         shift(newIndex, currentItem);
}
public int findSpot(currentItem) {
   for (sorted list)
      if (spot found) return
}
public void shift(newIndex, currentItem) {
   for (i = currentItem > newIndex)
      item[i+1] = item[i]
   item[newIndex] = currentItem
}
```

Worst case runtime?	$O(n^2)$
Best case runtime?	$O(n)$
Average runtime?	$O(n^2)$
Stable?	Yes
In-place?	Yes

Heap Sort

https://www.youtube.com/watch?v=Xw2D9aJRBY4

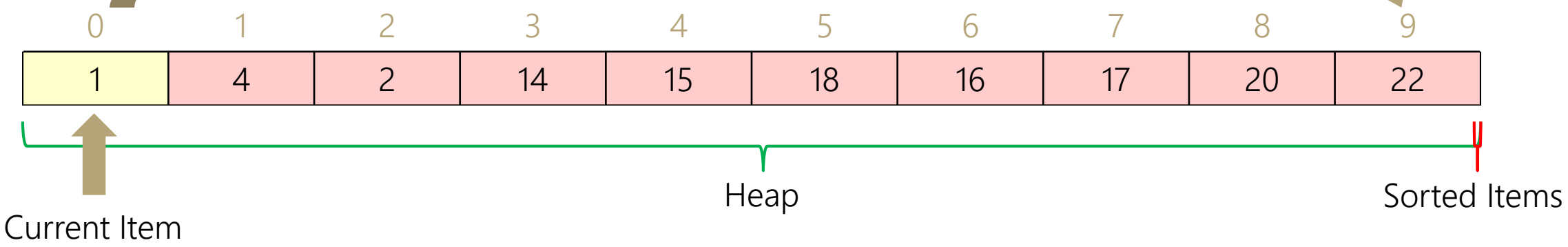
1. run Floyd's buildHeap on your data
2. call removeMin n times

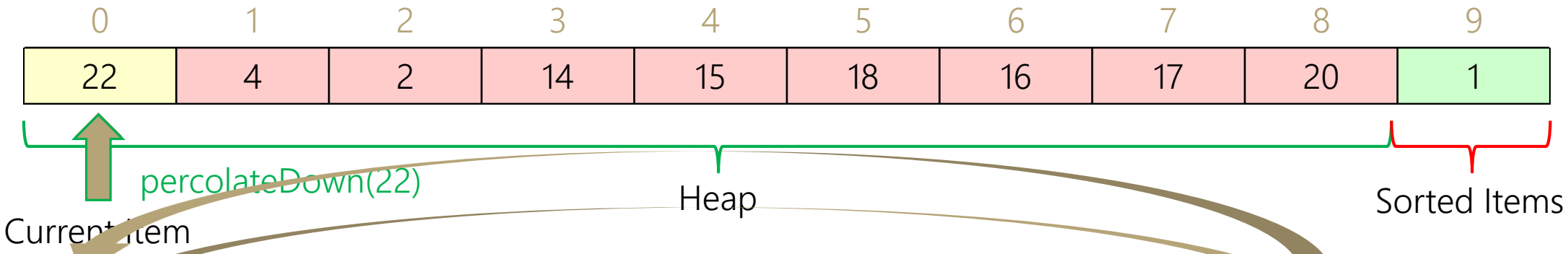


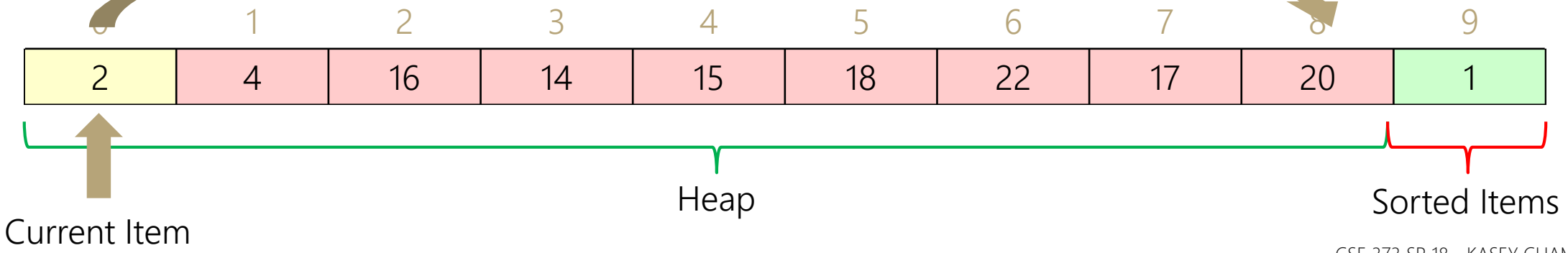
```
public void heapSort(collection) {
   E[] heap = buildHeap(collection)
   E[] output = new E[n]
   for (n)
      output[i] = removeMin(heap)
}
```

Worst case runtime?	O(nlogn)
Best case runtime?	O(nlogn)
Average runtime?	O(nlogn)
Stable?	No
In-place?	No

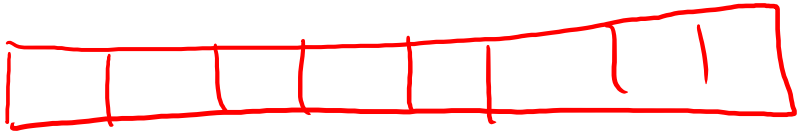
In Place Heap Sort

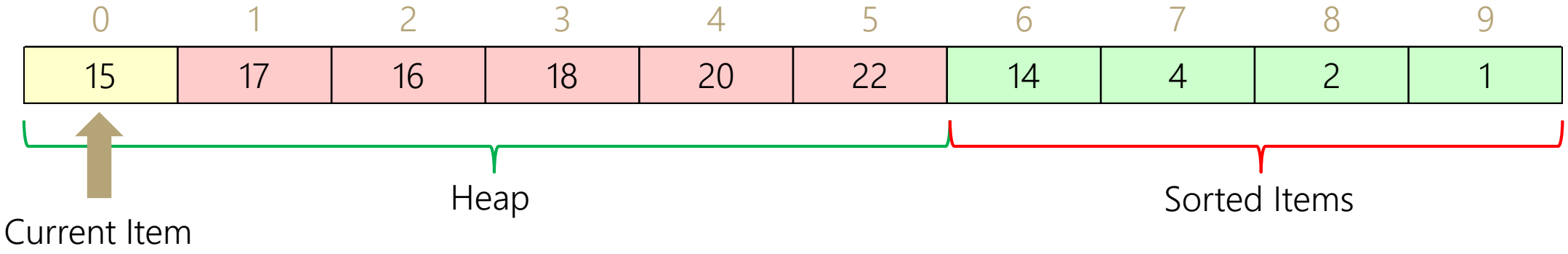






In Place Heap Sort






```
public void inPlaceHeapSort(collection) {
   E[] heap = buildHeap(collection)
   for (n)
      output[n - i - 1] = removeMin(heap)
}
```

Complication: final array is reversed!

- Run reverse afterwards (O(n))
- Use a max heap
- Reverse compare function to emulate max heap

Worst case runtime?	O(nlogn)
Best case runtime?	O(nlogn)
Average runtime?	O(nlogn)
Stable?	No
In-place?	Yes

Divide and Conquer Technique

$\boxed{2T(\frac{n}{2})} + n$

divide! conquer!

1. Divide your work into smaller pieces recursively
   - Pieces should be smaller versions of the larger problem
2. Conquer the individual pieces
   - Base case!
3. Combine the results back up recursively

```
divideAndConquer(input) {
   if (small enough to solve)
      conquer, solve, return results
   else
      divide input into a smaller pieces
      recurse on smaller piece
      combine results and return
}
```

← if size 1 return list

← divide list in pieces, sort recursively

← combine sorted lists

Merge Sort

https://www.youtube.com/watch?v=XaqR3G_NVoo

Divide

0	1	2	3	4	5	6	7	8	9
8	2	91	22	57	1	10	6	7	4

0	1	2	3	4
8	2	91	22	57

5	6	7	8	9
1	10	6	7	4

Conquer

0
8

0
8

Combine

0	1	2	3	4
2	8	22	57	91

5	6	7	8	9
1	4	6	7	10

0	1	2	3	4	5	6	7	8	9
1	2	4	6	7	8	10	22	57	91

Merge Sort

$T(1) = 1$

```
mergeSort(input) {
    if (input.length == 1)
        return
    else
        smallerHalf = mergeSort(new [0, ..., mid])
        largerHalf = mergeSort(new [mid + 1, ...])
        return merge(smallerHalf, largerHalf)
}
```

$T(1)=1$

$T(n) = 2T(\frac{n}{2}) + n$

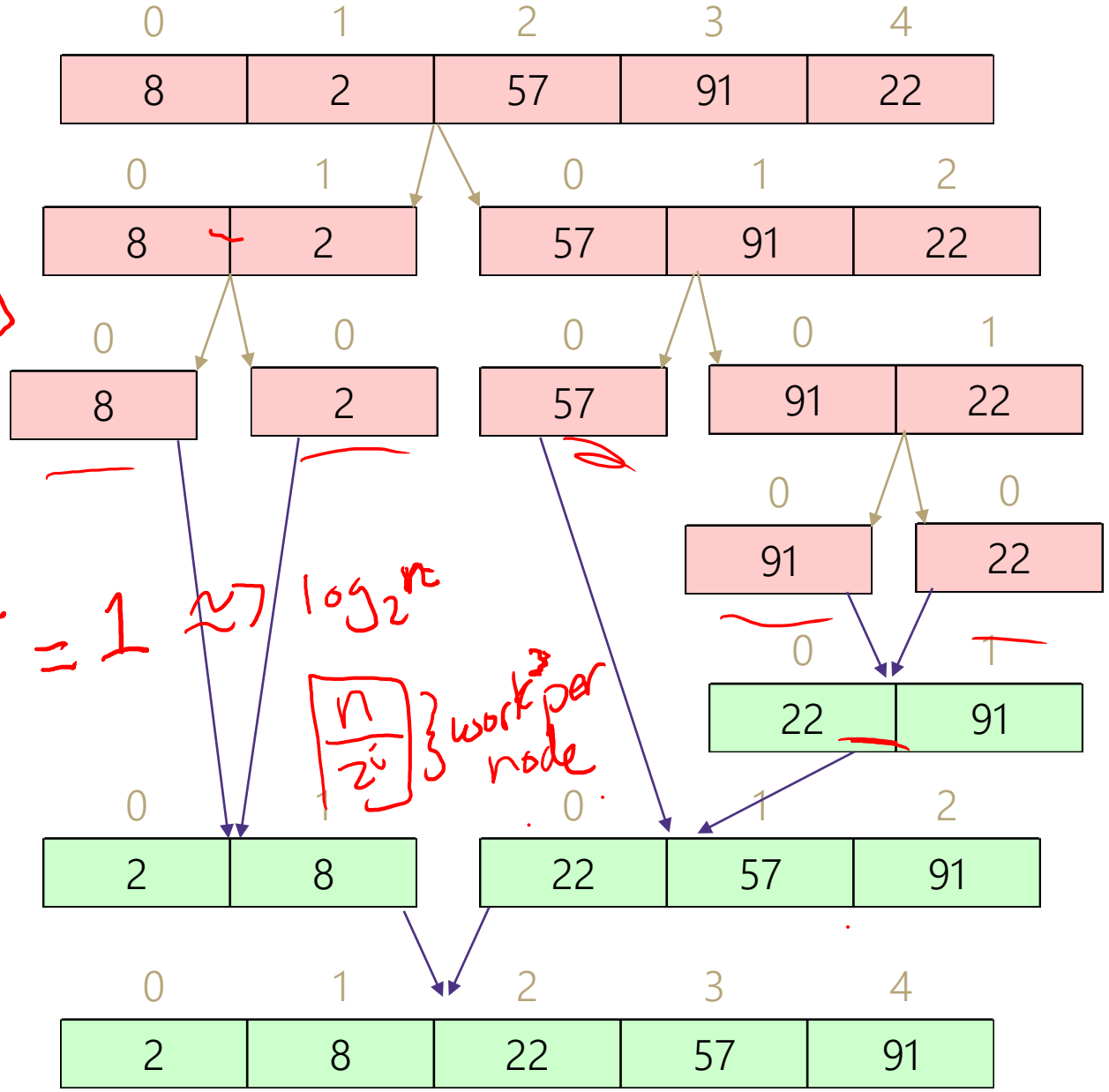
Worst case runtime? $O(n \log n)$

Best case runtime? $T(n) = \begin{cases} 1 \text{ if } n <= 1 \\ 2T(n/2) + n \text{ otherwise} \end{cases}$

Average runtime?

Stable? Yes

In-place? No



$\frac{n}{2^i} = 1 \rightsquigarrow \log_2 n$

$\boxed{\frac{n}{2^i}}$ } work per node

work per layer $= \underbrace{2^i}_{\#\text{ of nodes}} \cdot \underbrace{\frac{n}{2^i}}_{\text{work/node}} = n$

Merge Sort Optimization

Use just two arrays – swap between them

Quick Sort

https://www.youtube.com/watch?v=ywWBy6J5gz8

Divide

0	1	2	3	4	5	6	7	8	9
8	2	91	22	57	1	10	6	7	4

0	1	2	3	4
2	1	6	7	4

0
8

0	1	2	3
91	22	57	10

Conquer

0
6

Combine

0
6

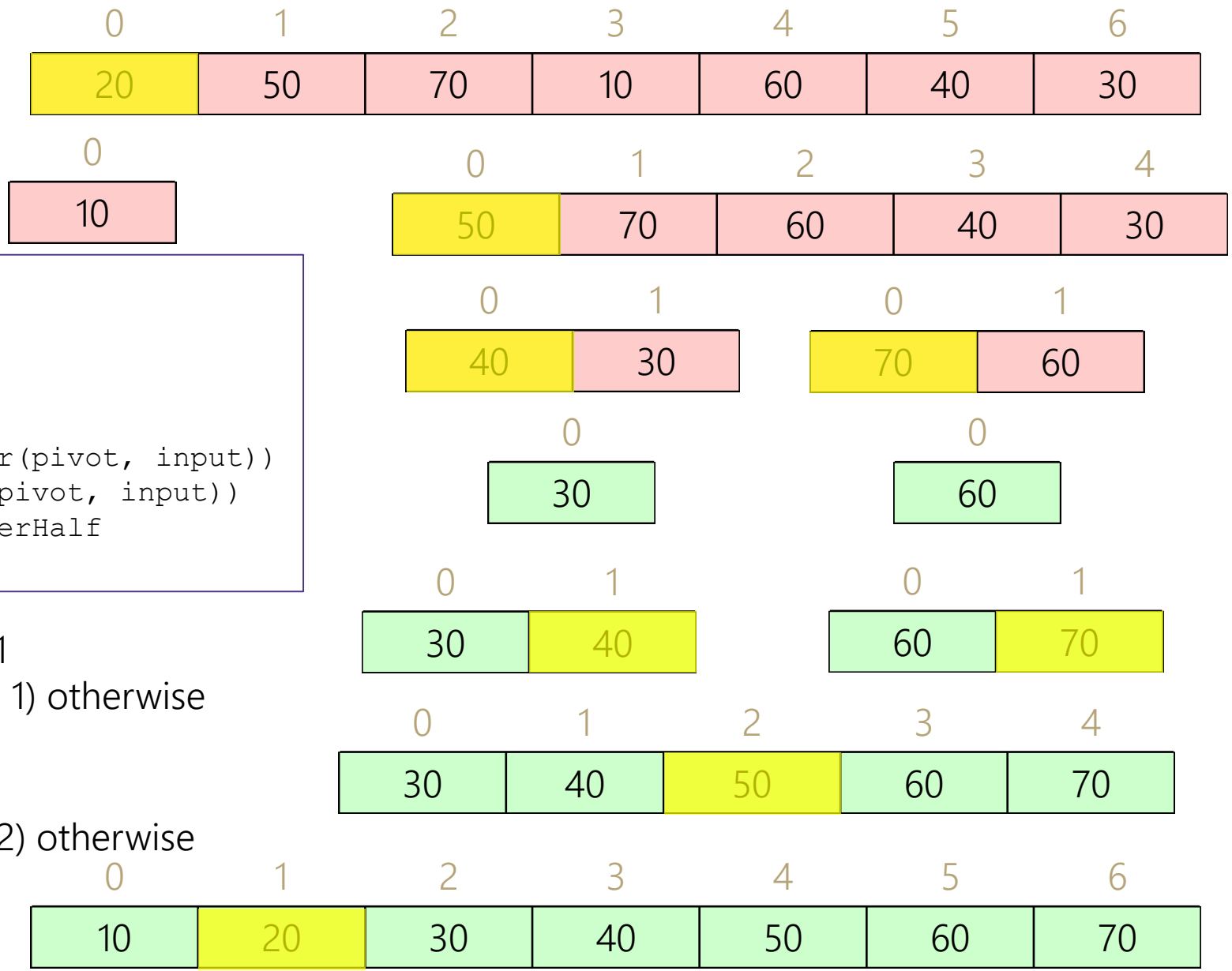
0	2	3	4
2	22	57	91

0
8

5	6	7	8	9
1	4	6	7	10

0	1	2	3	4	5	6	7	8	9
1	2	4	6	7	8	10	22	57	91

Quick Sort




```
quickSort(input) {
   if (input.length == 1)
      return
   else
      pivot = getPivot(input)
      smallerHalf = quickSort(getSmaller(pivot, input))
      largerHalf = quickSort(getBigger(pivot, input))
      return smallerHalf + pivot + largerHalf
}
```

Worst case runtime? $T(n) = \begin{cases} 1 \text{ if } n <= 1 \\ n + T(n-1) \text{ otherwise} \end{cases}$

Best case runtime?

Average runtime? $T(n) = \begin{cases} 1 \text{ if } n <= 1 \\ n + 2T(n/2) \text{ otherwise} \end{cases}$

Stable? No

In-place? No

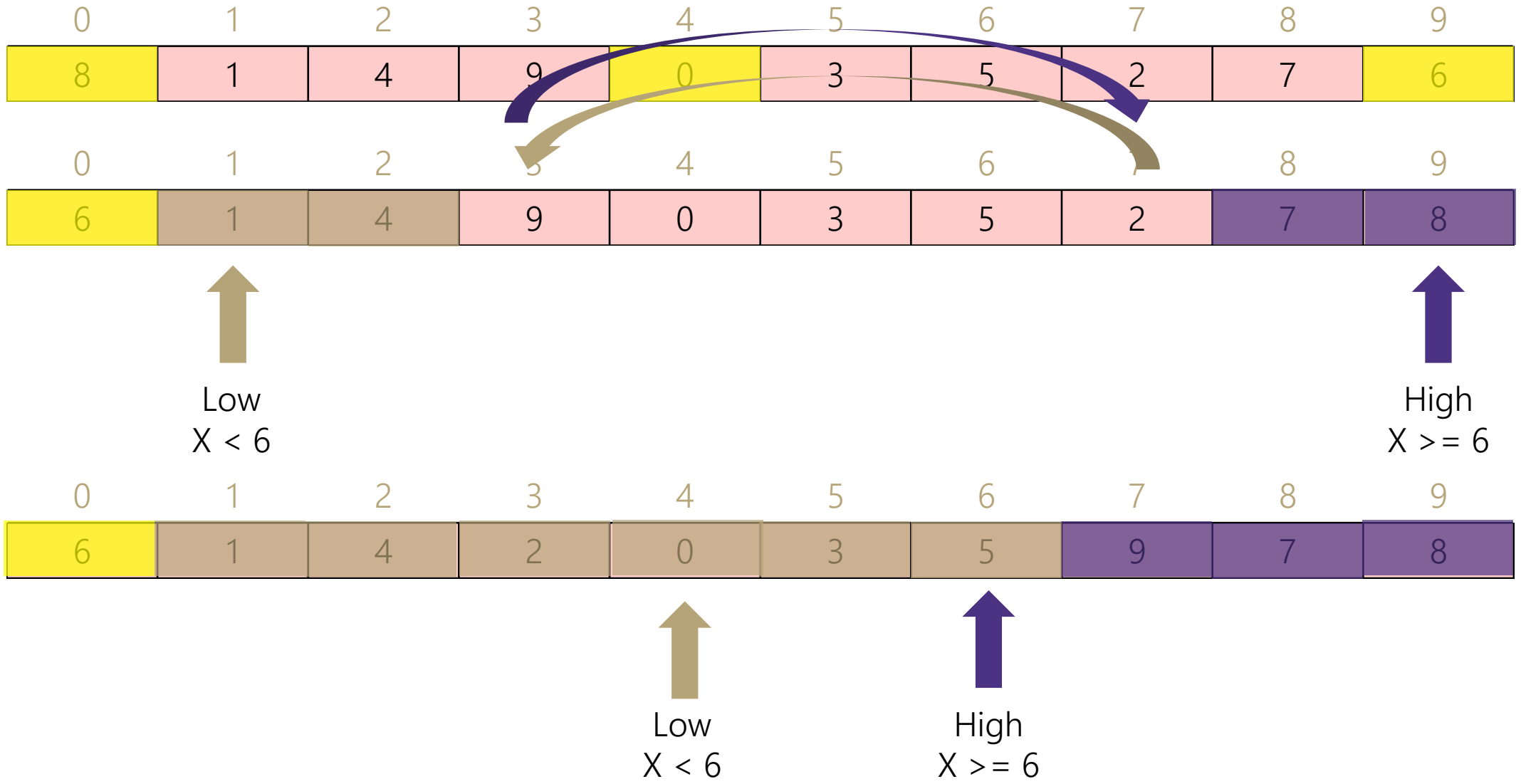
Can we do better?

Pick a better pivot

- Pick a random number
- Pick the median of the first, middle and last element

Sort elements by swapping around pivot in place

Better Quick Sort



Announcements

Project 1 (Calculator) is Due Tonight! Use “SUBMIT” as tag.

HW3 (Individual Assignment) will be assigned this weekend

- Due Sunday 7/22
- Make sure you know how to do it before the midterm! It’s the best midterm review

Come to Class Next Week:

- Monday: Midterm Review – Going from Diagrams to Code
- Wednesday: Software Engineering – Deep Dive into Git, Pair Programming, and Testing
- Friday: Midterm Exam! – Review materials and practice midterms on website (this evening).

More Announcements

If you are applying to the CSE major, send me an e-mail reminding me of our interactions

Office hours immediately after class – follow me!