## Section 05: Solutions

## 1. Stacks and Queues

Consider a sequence of characters and the task is to reverse the sequence. Is it beneficial to use a stack or a queue to perform this task? Assume that stacks and queues are implemented using linked lists and each node in the linked list stores a character.

## Solution:

A stack is preferred over a queue. This is because we can read the sequence of characters from left to right, pushing a character at a time into the stack. After pushing the sequence, we can pop elements from the stack until empty. It is easy to see that the order in which the elements are popped is exactly the reversal we desire.

## 2. Asymptotic Analysis

For each of the following, choose a $c$ and $n_{0}$ which show $f(n) \in \mathcal{O}(g(n))$. Explain why your values of $c$ and $n_{0}$ work.
(a) $f(n)=5000 n^{2}+6 n \sqrt{n}$ and $g(n)=n^{3}$

## Solution:

We will find a $c$ and $n_{0}$ such that $5000 n^{2}+6 n \sqrt{n} \leq c n^{3}$ for all values of $n \geq n_{0}$.
We have,

$$
\begin{aligned}
5000 n^{2}+6 n \sqrt{n} & \leq 5000 n^{2}+6 n^{2} & & \text { when } n \geq 1 \\
5000 n^{2}+6 n^{2} & \leq 5006 n^{2} & & \text { for all } n \\
5006 n^{2} & \leq 5006 n^{3} & & \text { for all } n \geq 1
\end{aligned}
$$

Therefore, we can pick $c=5006$ and $n_{0}=1$, which implies that $f(n) \in \mathcal{O}(g(n))$.
(b) $f(n)=2^{n}$ and $g(n)=3^{n}$

## Solution:

As before, we must find a $c$ and $n_{0}$ such that $2^{n} \leq c 3^{n}$ for all $n \geq n_{0}$.
Since $2<3,2^{n}<3^{n}$ for every $n \geq 1$. So, we can pick $c=1$ and $n_{0}=1$.

## 3. Recurrences

Solve these recurrences (give a Big-Theta bound). If the master theorem is applicable, state which case you used. If you use unrolling or the tree method, show your work.
(a)

$$
T(n)= \begin{cases}1 & \text { if } n=1 \\ T(n / 2)+n^{2} & \text { otherwise }\end{cases}
$$

## Solution:

Case 1 of the Master Theorem applies and hence $T(n)=\Theta\left(n^{2}\right)$.
(b)

$$
T(n)= \begin{cases}1 & \text { if } n=1 \\ 2 \cdot T(n / 4)+\sqrt{n} & \text { otherwise }\end{cases}
$$

Solution:
Case 2 of the Master Theorem applies and hence $T(n)=\Theta(\sqrt{n} \log n)$.
(c)

## 4. AVL/BST

Insert $\{6,5,4,3,2,1,10,9,8,6,7\}$ into an initially empty AVL tree.

## Solution:



## 5. Heaps

Insert $\{6,5,4,3,2,1,10,9,8,6,7\}$ into an initially empty min-heap. Write down the final heap as an array. Solution:

| 1 | 3 | 2 | 6 | 4 | 5 | 10 | 9 | 8 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## 6. Hash tables

(a) Consider the following sequence of numbers.

$$
6,29,41,34,10,64,50
$$

Suppose the hash function is $h(k)=2 k$. Insert each number into the following hash tables and draw what their internal state looks like:
(i) A hash table that uses linear probing, with internal capacity 10. Do not worry about resizing.

## Solution:

| 1 |  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 64 | 6 | 41 | 50 |  |  |  | 29 | 34 |

(ii) A hash table that uses quadratic probing, with internal capacity 10. Do not worry about resizing. Solution:

| 1 |  | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 50 | 6 | 41 |  |  |  | 64 | 29 | 34 |

