Name: Key

Email address: ____________________

CSE 373 Autumn 2010: Midterm #1

(closed book, closed notes, NO calculators allowed)

Instructions: Read the directions for each question carefully before answering. We may give partial credit based on the work you **write down**, so if time permits, show your work! Use only the data structures and algorithms we have discussed in class or which were mentioned in the book so far.

Note: For questions where you are drawing pictures, please circle your final answer for any credit.

Good Luck!

Total: 75 points. Time: 50 minutes.

Question	Max Points	Score
1	16	
2	8	
3	15	
4	4	
5	6	
6	8	
7	18	
Total	75	

1. (16 pts) **Big-O**
For each of the functions *f(N)* given below, indicate the tightest bound possible (in other words, giving $O(2^N)$ as the answer to every question is not likely to result in many points). Unless otherwise specified, all logs are base 2. **You MUST choose your answer from the following** (not given in any particular order), each of which could be re-used (could be the answer for more than one of a) – h)):

> $O(N^2)$, $O(N^{½})$$O(N^3 \log N)$, $O(N \log N)$, $O(N)$, $O(N^2 \log N)$, $O(N^5)$, $O(2^N)$, $O(N^3)$, $O(\log N)$, $O(1)$, $O(N^4)$, $O(N^{12})$$O(N^N)$, $O(N^6)$, $O(N^8)$, $O(N^9)$, $O(N^{10})$

You do not need to explain your answer.

a) $f(N) = (1/2)(N \log N) + (\log N)^2$ $O(N \log N)$

b) $f(N) = N^2 \cdot (N + N \log N + 1000)$ $O(N^3 \log N)$

c) $f(N) = N^2 \log N + 2^N$ $O(2^N)$

d) $f(N) = ((1/2)(3N + 5 + N))^4$ $O(N^4)$

e) $f(N) = (2N + 5 + N^4) / N$ $O(N^3)$

f) $f(N) = \log_{10}(2^N)$ $O(N)$

g) $f(N) = N! + 2^N$ $O(N^N)$

h) $f(N) = (N \cdot N \cdot N \cdot N + 2N)^2$ $O(N^8)$

2. (8 pts) **Big-Oh and Run Time Analysis:** Describe the worst case running time of the following pseudocode functions in Big-Oh notation in terms of the variable n. ***Showing your work is not required*** (although showing work ***may*** allow some partial credit in the case your answer is wrong – don't spend a lot of time showing your work.). You MUST choose your answer from the following (not given in any particular order), each of which could be re-used (could be the answer for more than one of I. – IV.):

$O(n^2)$, $O(n^3 \log n)$, $O(n \log n)$, $O(n)$, $O(n^2 \log n)$, $O(n^5)$, $O(2^n)$, $O(n^3)$, $O(\log n)$, $O(1)$, $O(n^4)$, $O(n^n)$

I.
```
void silly(int n) {
    for (int i = 0; i < n; ++i) {
        j = n;
        while (j > 0) {
            System.out.println("j = " + j);
            j = j - 2;
        }
    }
}
```

Runtime: $O(n^2)$

II.
```
void silly(int n, int x, int y) {
    for (int k = n; k > 0; k--)
        if (x < y + n) {
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < i; ++j)
                    System.out.println("y = " + y);
        } else {
            System.out.println("x = " + x);
        }
}
```

$O(n^3)$

III.
```
void silly(int n) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j)
            System.out.println("j = " + j);
        for (int k = 0; k < i; ++k) {
            System.out.println("k = " + k);
            for (int m = 0; m < 100; ++m)
                System.out.println("m = " + m);
        }
    }
}
```

$O(n^2)$

IV.
```
int silly(int n, int m) {
    if (m < 2) return m;
    if (n < 1) return n;
    else if (n < 10)
        return silly(n/m, m);
    else
        return silly(n - 1, m);
}
```

$O(n)$

3. (15 pts total) **Trees**.

a) (4 pts) What is the minimum and maximum number of nodes in an **AVL tree of height 6?** (Hint: the height of a tree consisting of a single node is 0) ***Give an exact number*** for both of your answers – not a formula.

Minimum = 33 $\quad (S(h) = S(h-1) + S(h-2) + 1)$

Maximum = 127 $\quad (2^{h+1} - 1 = 2^{7} - 1 = 128 - 1)$

b) (3 pts) Give traversals of the tree shown at the bottom of this page:

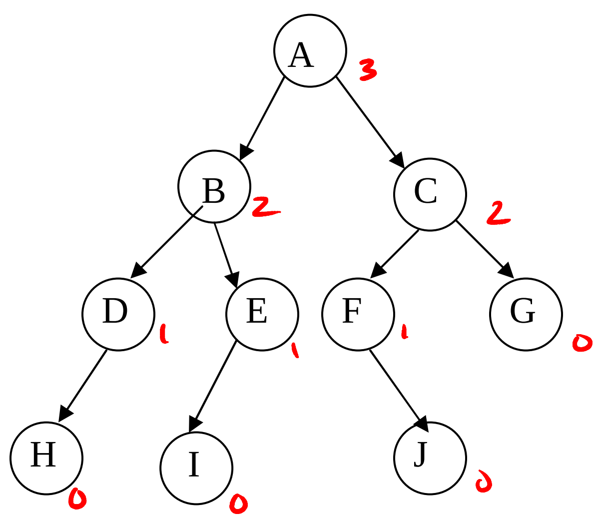
Pre-Order: A B D H E I C F J G

Post-Order: H D I E B J F G C A

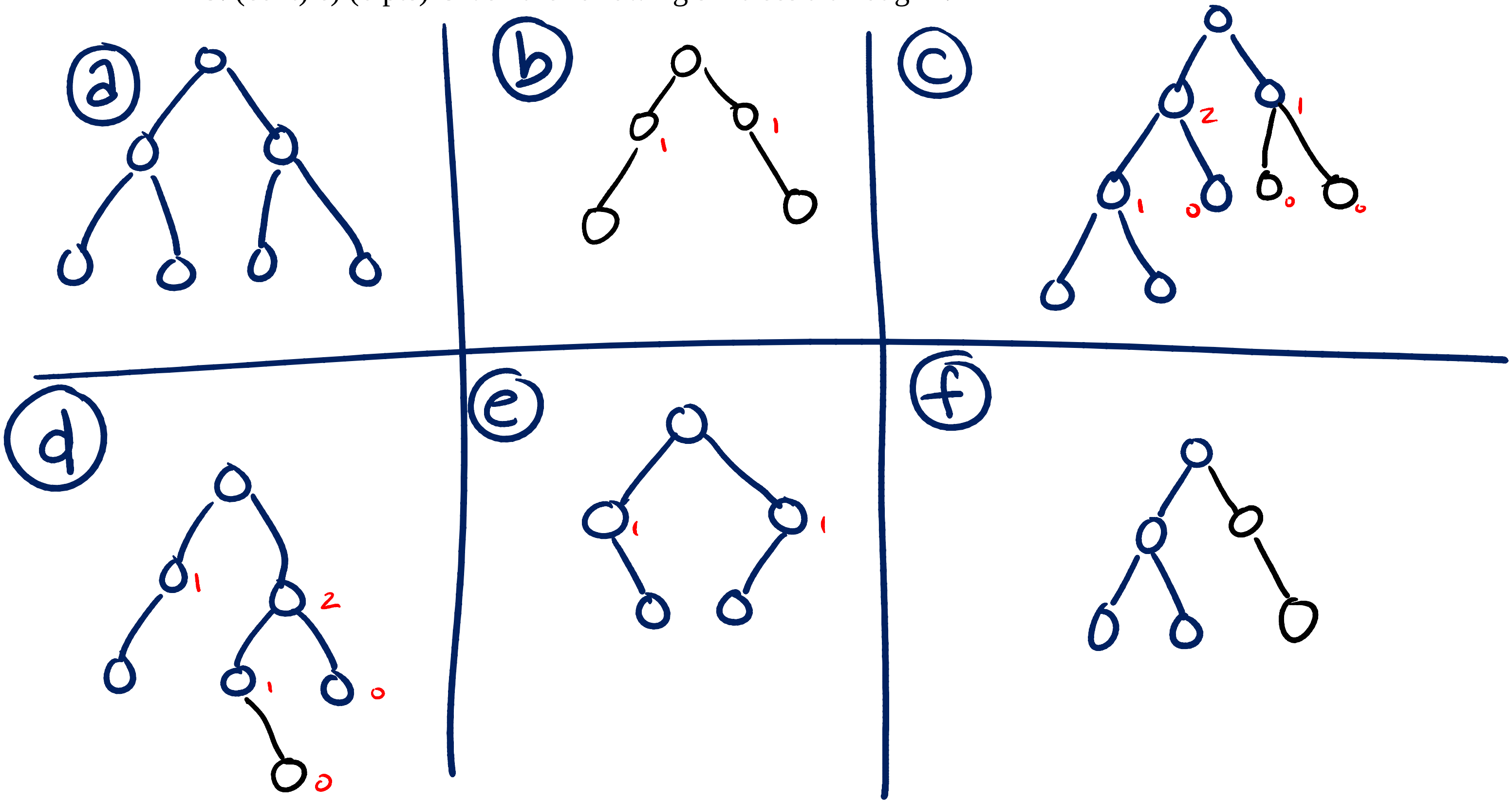
In-Order: H D B I E A F J C G

c) (1 pt) What is the **depth** of node F in the tree shown below: 2

d) (1 pt) Is it AVL balanced (ignore the values, only look at the shape): YES / NO



3. (cont) e) (6 pts) Given the following six trees a through f:



List the letters of all of the trees that have the following properties: (Note: It is possible that none of the trees above have the given property, it is also possible that some trees have more than one of the following properties.)

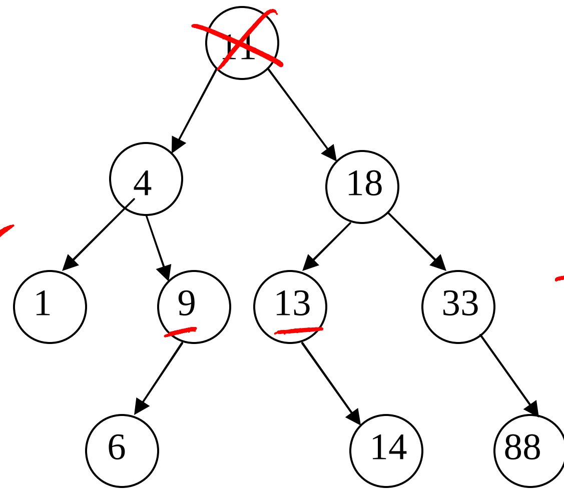
Full: a, c

Complete: a, c

AVL balanced: a, b, c, d, e, f

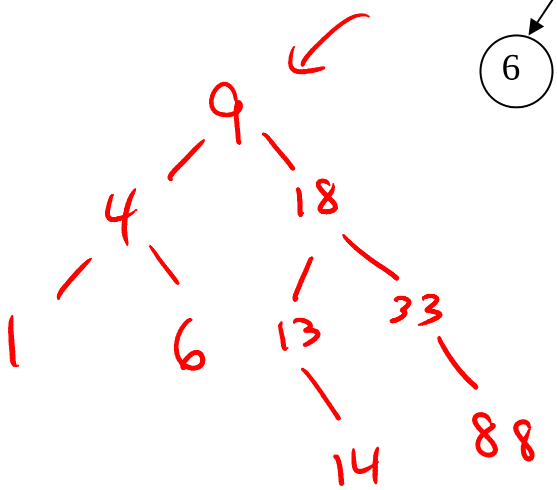
4. (4 pts total) **Binary Search Trees & AVL Trees**

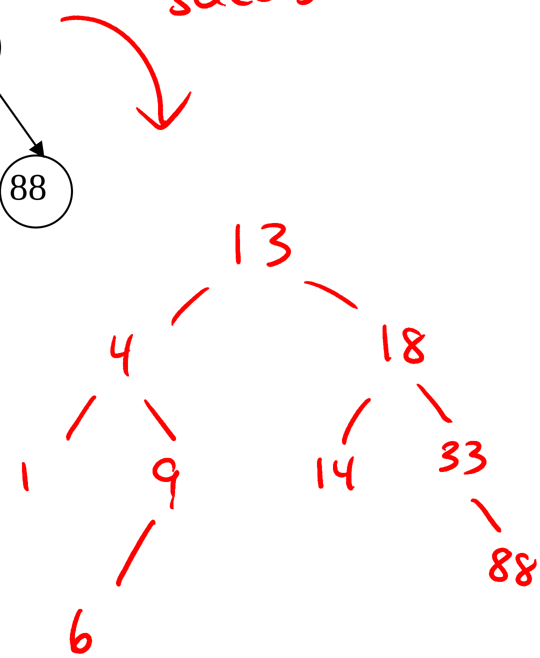
a) (2 pts) Given the **binary search tree** shown below. Draw what the tree would look like after deleting the value 11. Use one of the methods for deleting described in class or in the book – do NOT use lazy deletion.



replace w. predecessor

replace w. successor



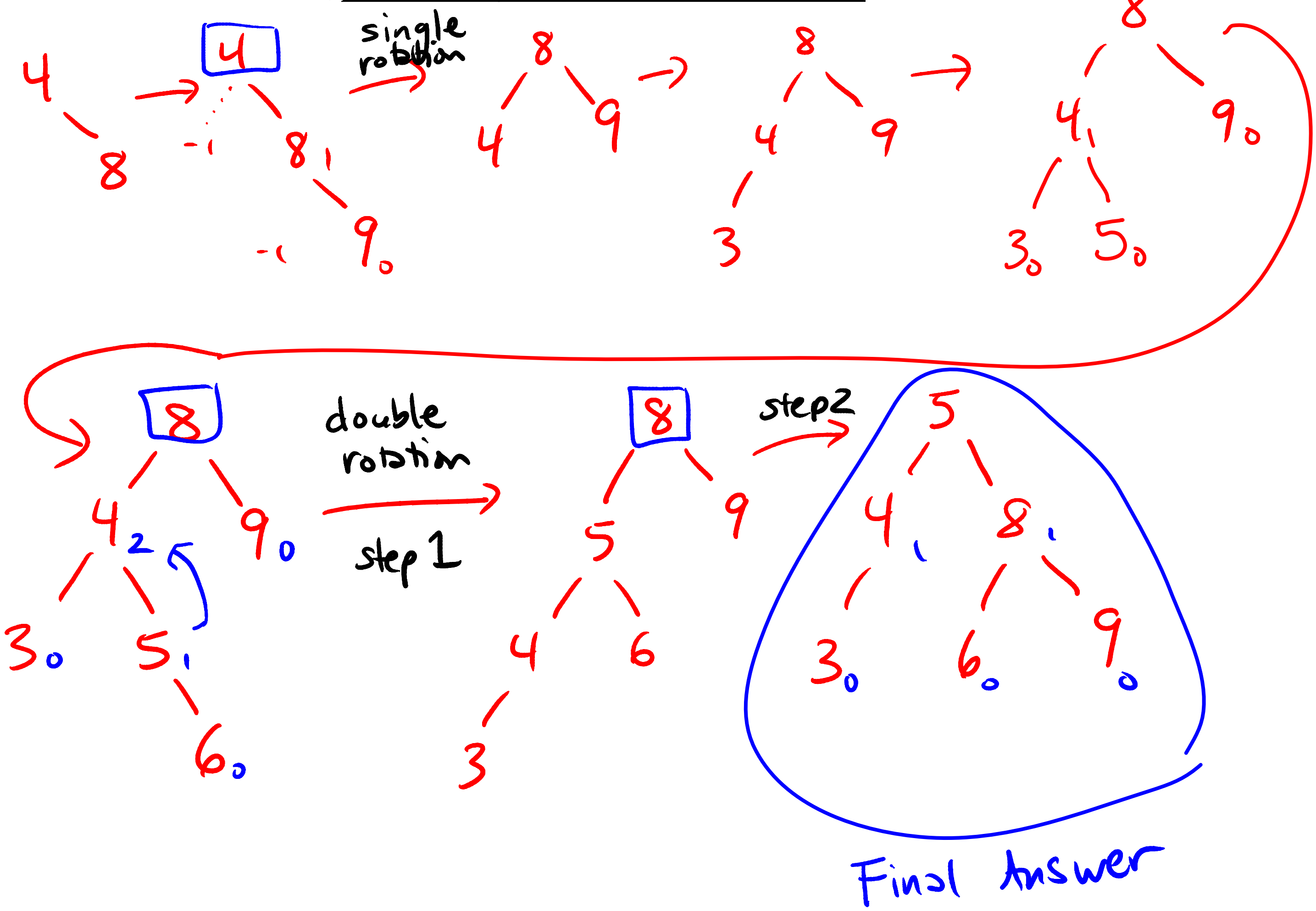


b) (2 pts) You are given an **<u>AVL tree</u>** of height 6. The minimum and maximum number of *rotations* we might have to do when doing an insert is: (**<u>Give an exact number, not a formula.</u>** A single rotation = 1 rotation, a double rotation = 1 rotation)

Minimum = 0

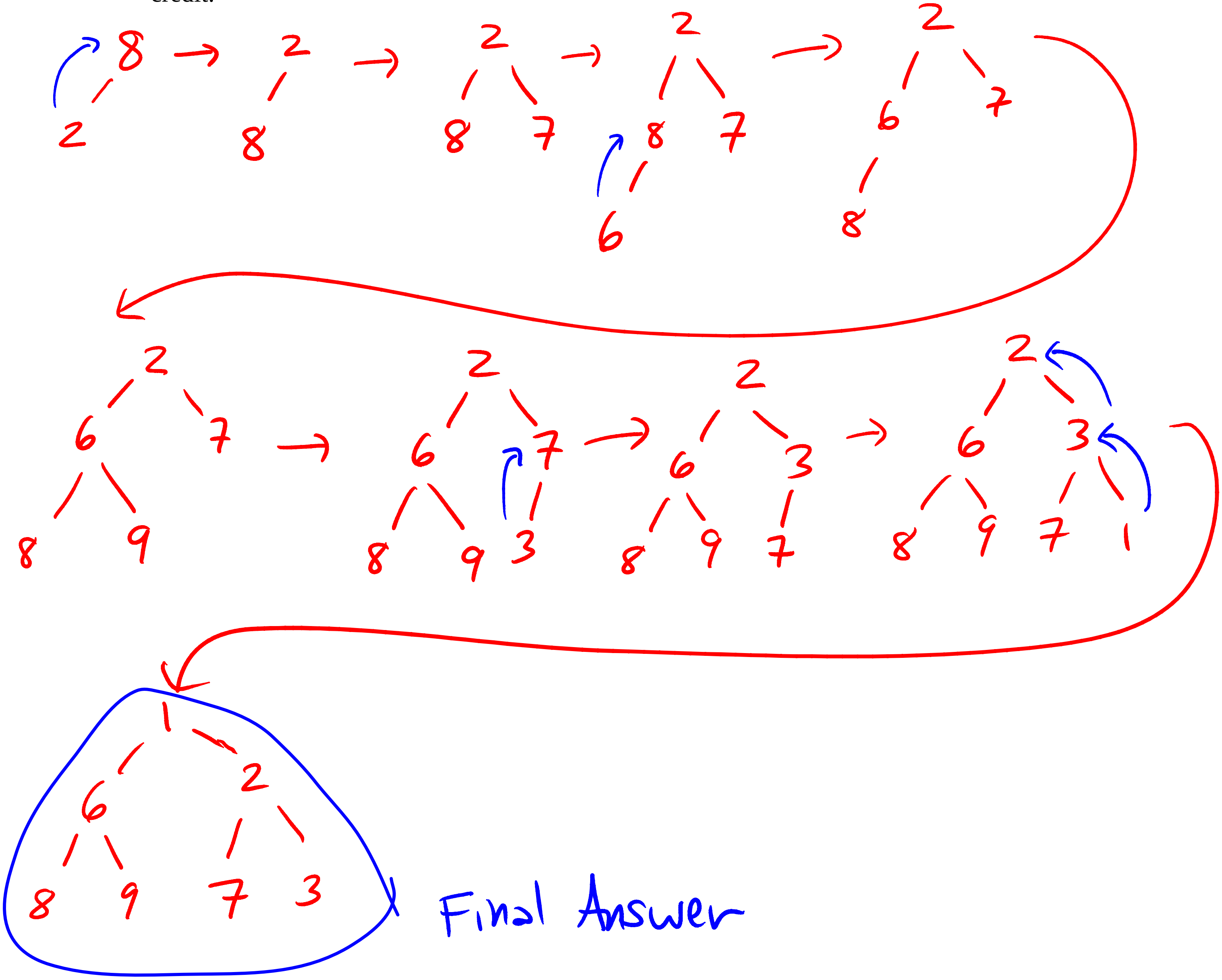
Maximum = 1

5. (6 pts) **AVL Trees** Draw the AVL tree that results from inserting the keys: 4, 8, 9, 3, 5, 6 in that order into an initially empty AVL tree. You are only required to show the final tree, although drawing intermediate trees may result in partial credit. If you draw intermediate trees, ***please circle your final tree for ANY credit.***



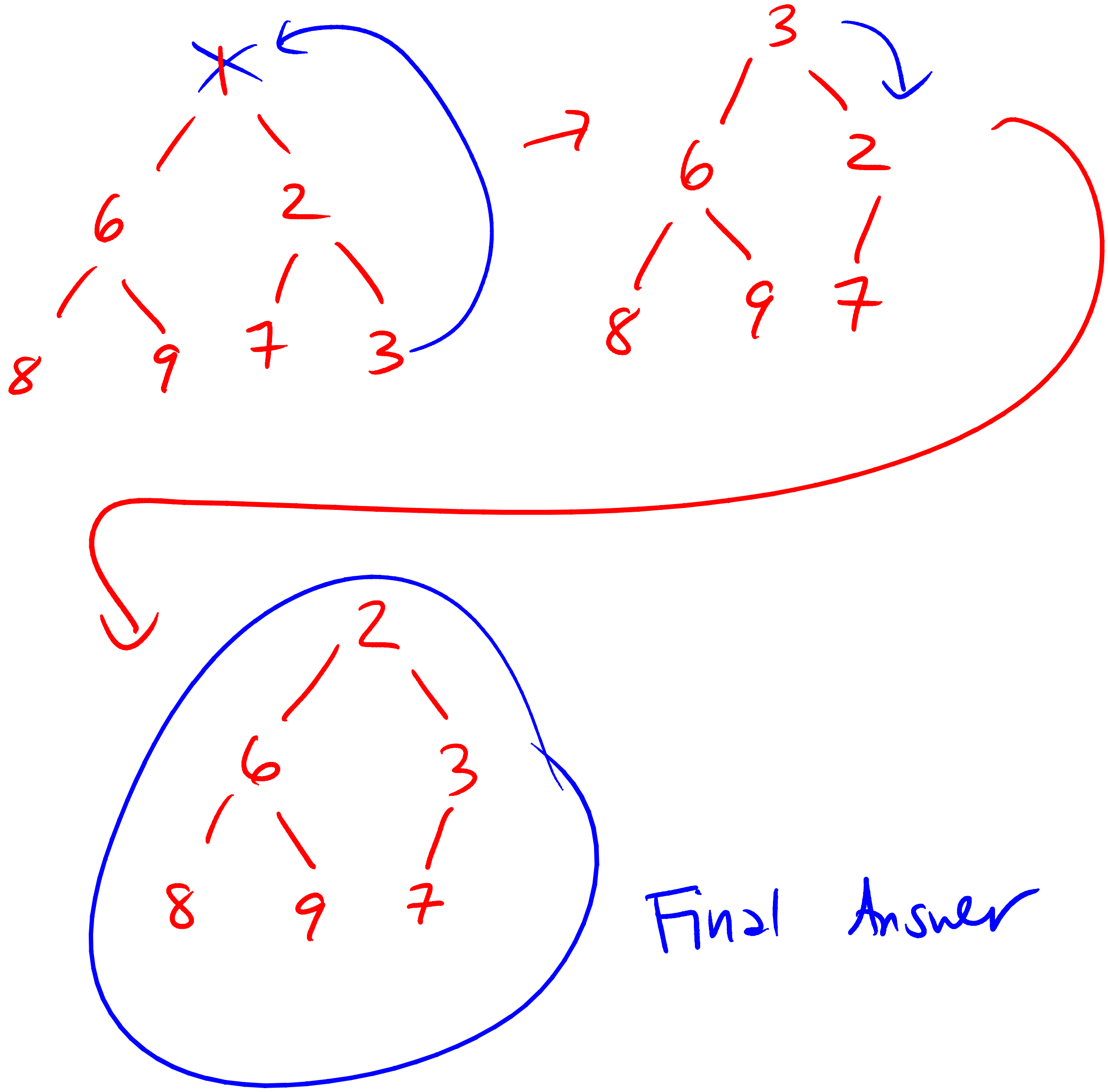
6. (8 pts total) **Binary Min Heaps**

(a) [6 points] Draw the binary min heap that results from inserting ~~8, 2, 7, 6, 9~~, 3, 1 in that order into an initially empty binary min heap. *You do not need to show the array representation of the heap*. You are only required to show the final tree, although drawing intermediate trees may result in partial credit. If you draw intermediate trees, please **circle your final result** for any credit.



6. **(cont.)**

(b) [2 points] Draw the result of one deletemin call on your heap drawn at the end of part (a).



7. (18 pts) **Algorithms & Running Time Analysis:**

- **Describe the most time-efficient way to implement the operations listed below**. Assume no duplicate values and that you can implement the operation as a member function of the class – with access to the underlying data structure.
- Then, give the tightest possible upper bound for the ***worst case*** running time for each operation in terms of *N*. **For any credit, you must explain *why* it gets this worst case running time.** You must choose your answer from the following (not listed in any particular order), each of which could be re-used (could be the answer for more than one of a) -f)).

$O(N^2)$, $O(N^{1/2})O(N^3 \log N)$, $O(N \log N)$, $O(N)$, $O(N^2 \log N)$, $O(N^5)$, $O(2^N)$, $O(N^3)$, $O(\log N)$, $O(1)$, $O(N^4)$, $O(N^{12})O(N^N)$, $O(N^6)$, $O(N^8)$, $O(N^9)$

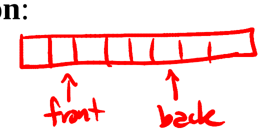
a) Pushing a value onto a stack containing N values, implemented as a linked list. **Explanation:**

① Create a new node p, ② set p.next → top, ③ set top → p. All of these are constant time operations.

a) O(1)

b) Enqueue a value onto a queue containing N values implemented as a circular array (as described in class). (Assume the array is size N+5.) **Explanation**:

① Put new value in array[back]
② back = (back+1) % (N+5)
(All constant time operations.)



b) O(1)

c) Deleting the minimum value in a binary min heap of size N. **Explanation:**

① Remove the root, ② Put rightmost value on bottom row into root position, ③ Percolate down – comparing vs. children + swap w. smaller child if needed. Height = O(logN) = max # of swaps

c) O(logN)

d) Given a binary search tree containing N integers, create an AVL tree containing the same values. You should not destroy the original BST in the process. **Explanation:**

Traverse the BST (in any order), as you visit a node, insert that value into the AVL tree. Each AVL insert is O(log N) due to max height of tree. You are doing N inserts so O(N log N).

d) O(N logN)

e) Printing out the values stored in all of the *leaves* of a perfect BST containing N values in ascending order. **Explanation:** Do an inorder traversal, at each node, if both of its children are null, print it out. Checking for null is constant time operation. Must visit each node once → O(N) for traversal.

e) O(N)

f) Given an AVL tree containing N positive integers, print out all the even values contained in the tree in *descending* order (e.g. 12, 8, 6, 2). Be sure to explain how you will get descending order. **Explanation:** Do a traversal as follows:

```
trav(n node){
  trav(right)
  if (n % 2 == 0) print n
  trav(left)
}
```

Visit each node once, O(N).

f) O(N)