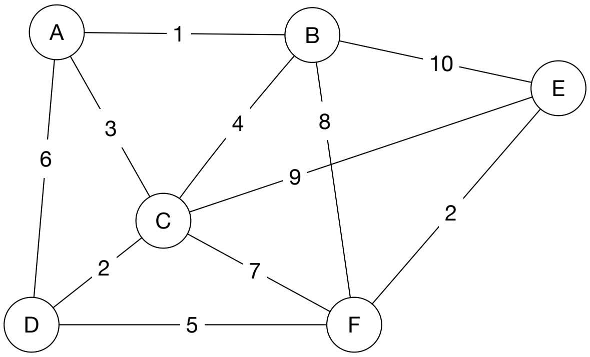
Name & Student Number:

1. Answer each of the following questions as True or False.
   (a) A MST contains a cycle. ________________
   (b) If we remove an edge from a MST, the resulting subgraph is still a MST. ________________
   (c) If we add an edge to a MST, the resulting subgraph is still a MST.________________
   (d) If there are V vertices in a given graph, a MST of that graph contains $|V| - 1$ edges.________________
   (e) Every MST is a sparse graph.________________
2. Following is the pseudocode for Kruskal's algorithm to find a MST.

```
function Kruskal(Graph G)
    initialize each vertex to be a component
    sort all edges by weight
    for each edge (u, v) in sorted order do
        if u and v are in different components then
            add edge (u,v) to the MST
            update u and v to be in the same component
        end if
    end for
end function
```

(a) Execute Kruskal's algorithm on the following graph. Fill the table.



Step	Components	Edge	Include?
1	{A} {B} {C} {D} {E} {F}	A, B	Yes
2	{A, B} {C} {D} {E} {F}	B, C	
3			
4			
5			
6			
7			
8			
9			
10			
11			

(b) In this graph there are 6 vertices and 11 edges, and the for loop in the above pseudocode iterates 11 times, a few more times after the MST is found. How would you optimize the pseudocode so the for loop terminates early, as soon as a valid MST is found. Annotate the given pseudocode to add/edit lines.

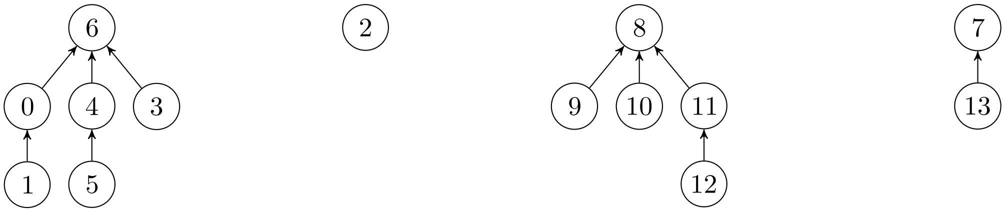


Figure 1: Disjoin-set. Rank of trees in the forest (from left): 2, 0, 2, 1.

3. Consider the disjoin-set shown in Figure **??**
   What would be the result of the following calls on union if we add the "union-by-rank" optimization. Draw the forest at each stage with corresponding ranks for each tree
   i. `union(2, 13)`

   ii. `union(4, 12)`

   iii. `union(2, 8)`