Instructions: Read the directions for each question carefully before answering. We may give partial credit based on the work you write down, so if time permits, show your work! Use only the data structures and algorithms we have discussed in class or which were mentioned in the book so far.

Note: For questions where you are drawing pictures, please circle your final answer for any credit.

Good Luck!

Total: 72 points. Time: 50 minutes.

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1. (16 pts) Big-O
For each of the functions \( f(N) \) given below, indicate the tightest bound possible (in other words, giving \( O(2^N) \) as the answer to every question is not likely to result in many points). Unless otherwise specified, all logs are base 2. **You MUST choose your answer from the following** (not given in any particular order), each of which could be re-used (could be the answer for more than one of a) – h):

\[
O(N^2), O(N^{1/2}), O(N^{1/4}), O(\log^3 N), O(N), O(N^2 \log N), O(\log^8 N), O(N^5), O(2^N), O(N^3), O(N^8), O(\log^4 N), O(N \log^3 N), O(\log^2 N) O(\log N), O(1), O(N^3 \log N), O(N^4), O(N^N), O(N^6), O(N \log^2 N), O(\log \log N), O(\log N), O(N^3 \log N), O(N^4), O(N^N), O(N^6), O(N \log^2 N), O(\log \log N), O(\log^4 N), O(N \log N)
\]

You do not need to explain your answer.

a) \( f(N) = N^3 \cdot (4\log N - \log N) + (N^2)^2 \)  ________________

b) \( f(N) = \log_8(2^N) \)  ________________

c) \( f(N) = 500 N \log N + N^2 \log N \)  ________________

d) \( f(N) = (N \cdot (N + 1)^2) \)  ________________

e) \( f(N) = \log N^4 + \log^2 N \)  ________________

f) \( f(N) = N + \log N + N^{1/8} \)  ________________

g) \( f(N) = N^{1/4} + (\log^4 N)^2 \)  ________________

h) \( f(N) = N^3 \cdot N^2 + (N + 2N)^2 \)  ________________
2. (8 pts) **Big-Oh and Run Time Analysis:** Describe the worst case running time of the following pseudocode functions in Big-Oh notation in terms of the variable $n$. *Showing your work is not required* (although showing work *may* allow some partial credit in the case your answer is wrong – don’t spend a lot of time showing your work.). You MUST choose your answer from the following (not given in any particular order), each of which could be re-used (could be the answer for more than one of I. – IV.):

- $O(n^2)$
- $O(n^3 \log n)$
- $O(n \log n)$
- $O(n^2 \log n)$
- $O(2^n)$
- $O(n^3)$
- $O(\log n)$
- $O(1)$
- $O(n^4)$
- $O(n^6)$
- $O(n^8)$
- $O(n^7)$

**I.**

```java
int happy (int n, int m) {
    if (n < 10) return n;
    else if (n < 100)
        return happy (n - 2, m);
    else
        return happy (n/2, m);
}
```

**Runtime:**

**II.**

```java
void sunny (int n) {
    j = 0;
    while (j < n) {
        for (int i = 0; i < n; ++i) {
            System.out.println("i = " + i);
            for (int k = 0; k < i; ++k)
                System.out.println("k = " + k);
        }
        j = j + 1;
    }
}
```

**III.**

```java
void smiley (int n) {
    for (int i = 0; i < n * n; ++i) {
        for (int k = 0; k < i; ++k)
            System.out.println("k = " + k);
        for (int j = n; j > 0; j--)
            System.out.println("j = " + j);
    }
}
```

**IV.**

```java
void funny (int n, int x) {
    for (int k = 0; k < 100; ++k)
        if (x > 500) {
            for (int i = 0; i < n * k; ++i)
                for (int j = 0; j < n; ++j)
                    System.out.println("x = " + x);
        }
}
```
3. (19 pts total) Trees.

a) (4 pts) What is the minimum and maximum number of leaf nodes in a complete tree of height 6? (Hint: the height of a tree consisting of a single node is 0) 
Give an exact number (with no exponents) for both of your answers – not a formula.

Minimum =
Maximum =

b) (4 pts) What is the minimum and maximum number of nodes in a balanced AVL tree of height 6? Give an exact number (with no exponents) for both of your answers – not a formula.

Minimum =
Maximum =

c) (2 pts) What is the maximum number of leaf nodes in a full tree of height 5? 
Give an exact number (with no exponents) for your answer – not a formula.

Maximum =

d) (1 pt) What is the depth of node K in the tree shown below:

e) (1 pt) Give a Pre-Order traversal of the tree shown below:

f) (1 pt) Is it AVL balanced (ignore the values, only look at the shape):

YES / NO
3. (cont) g) (6 pts) Given the following six trees a through f:

List the letters of all of the trees that have the following properties: (Note: It is possible that none of the trees above have the given property, it is also possible that some trees have more than one of the following properties.)

Complete: ______________________

Full: ______________________

AVL balanced: ______________________

Perfect: ______________________
4. (11 pts total) Stacks – Please ANSWER PART (a) and PART (b).

(a) (8 pts) Given the stack interface from homework 1, write Java code that will determine which value in the stack is the largest and remove and return that value while otherwise leaving the remainder of the stack in its original order.

Thus if the stack1 contains values: (bottom 4 6 8 3 2 5 top) then a call to 
findAndRemoveLargest(stack1) would return the value 8 and leave stack1 as follows: (bottom 4 6 2 5 top)

In your solution, you may only declare extra DStacks or scalar variables (e.g. ints, doubles – not arrays or linked lists) if needed. You may not use other methods or classes from java collections. You can assume that any DStacks you use will never become full and that the provided stack will not contain duplicate values. Rather than throwing an exception, if the provided stack is empty findAndRemoveLargest(DStack) will return the value -373.

Unlike homework 1, assume that the only implementation of DStack that currently exists is the class LogStack. LogStack implements all of the operations of the DStack interface, but does them each at a cost of O(log N). If you need to create any temporary DStacks in your code, you will need to create new LogStacks. The interface for DStack is given below:

```
public interface DStack {
    // Returns true if stack is empty, false otherwise.
    public boolean isEmpty();

    // Adds d to the top of the stack.
    public void push(double d);

    // Returns and removes the value on top of stack. 
    // @throws EmptyStackException if stack is empty
    public double pop();

    // Returns (but does not remove) the value on top of stack. 
    // @throws EmptyStackException if stack is empty
    public double peek();
}
```

Please write your code for findAndRemoveLargest(DStack myStack) on the next page.

(b) (3 pts) Since the only implementation of DStack that currently exists is the class LogStack, all DStack parameters passed to findAndRemoveLargest(DStack myStack) must be LogStacks. Thus all stack operations (isEmpty, push, pop, peek) on any stacks in your code take time O(log n) each.

What is the worst case Big-O running time of your implementation of findAndRemoveLargest? For full credit give your answer in the most simplified form possible. (An explanation is not required, but could help provide partial credit. No credit if your solution is not mostly correct.)
public static double findAndRemoveLargest(DStack myStack) {
    // Add your code here:
}
5. (6 pts) **AVL Trees** Draw the AVL tree that results from inserting the keys: 1, 3, 7, 5, 6, 9 in that order into an initially empty AVL tree. You are only required to show the final tree, although drawing intermediate trees may result in partial credit. If you draw intermediate trees, please circle your final tree for any credit.
6. (12 pts) **Algorithms & Running Time Analysis:**

- **Describe the most time-efficient way to implement the operations listed below.** Assume no duplicate values and that you can implement the operation as a member function of the class – with access to the underlying data structure, including knowing the number of values currently stored (N).

- Then, give the tightest possible upper bound for the **worst case** running time for each operation in terms of N. **For any credit, you must explain why it gets this worst case running time.** You must choose your answer from the following (not listed in any particular order), each of which could be re-used (could be the answer for more than one of a) -f)).

  \[ O(N^2), O(N^{1.5}), O(N^3 \log N), O(N \log N), O(N), O(N^2 \log N), O(N^5), O(2^N), O(N^3), \]
  \[ O(\log N), O(1), O(N^6), O(N^{12})O(N^N), O(N^6), O(N^8), O(\log \log N) \]

a) Given a FIFO queue implemented as a linked list, find and remove all of the values greater than 10, leaving the rest of the queue in its original order. **Explanation:**

b) Given a binary search tree, print all of the odd values from largest to smallest. Be specific about how you would get values in order from largest to smallest. **Explanation:**
6. (continued) **Algorithms & Running Time Analysis:**

- **Describe the most time-efficient way to implement the operations listed below.** Assume no duplicate values and that you can implement the operation as a member function of the class – with access to the underlying data structure, including knowing the number of values currently stored (N).

- Then, give the tightest possible upper bound for the **worst case** running time for each operation in terms of \( N \). **For any credit, you must explain why it gets this worst case running time.** You must choose your answer from the following (not listed in any particular order), each of which could be re-used (could be the answer for more than one of a) -f)).

\[
O(N^2), O(N^{\sqrt{2}}), O(N^3 \log N), O(N \log N), O(N), O(N^2 \log N), O(N^5), O(2^N), O(N^3), O(\log N), O(1), O(N^5), O(N^{12}), O(N^{10}), O(N^6), O(N^8), O(\log \log N)
\]

c) Given a binary search tree, find which value is the **minimum** value and delete it. 
**Explanation:**

```

```

d) Given a FIFO queue implemented as a linked list currently containing N values, enqueue N more values so that when you are finished the queue will contain 2N values (Give the total time for enqueueing N more values). **Explanation:**

```

```