CSE 373: Data Structures and Algorithms

Lecture 10: AVL Trees

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Quarter: Summer 2017
Today

• Announcements
• BSTs continued (this time, bringing
  • buildTree
  • Balance Conditions
  • AVL Trees
  • Tree rotations
Announcements

• Reminder: homework 3 due Friday
• Homework 2 grades should come out today or tomorrow!
• Section
  • Will especially go over material from today (it’s especially tricky)
  • TAs can go over some of the tougher hw2 questions in section if you want/ask
Back to Binary Search Trees
let's consider `buildTree` (insert values starting from an empty tree)

insert values 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

- if inserted in given order, what is the tree?

- what big-O runtime for `buildTree` on this sorted input?

- is inserting in the reverse order any better?
buildTree for BST

Insert values 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

What we if could somehow re-arrange them

- median first, then left median, right median, etc.
  5, 3, 7, 2, 1, 4, 8, 6, 9

- What tree does that give us?

- What big-O runtime? $O(n \log n)$
Balancing Binary Search Trees
BST: Efficiency of Operations?

Problem: 
operations may be inefficient if BST is unbalanced

Worst-case running time:
• find, insert, delete \( O(n) \)
• buildTree \( O(n^2) \)
How can we make a BST efficient?

*Observation*

BST: the shallower, the better

*Solution:* Require a **Balance Condition** that

1. Ensures depth is always \( O(\log n) \)
2. Is efficient to maintain

- When we **build** the tree, make sure it’s balanced.
- **BUT**...Balancing a tree only at build time is insufficient.
- We also need to also **keep** the tree balanced as we perform operations.
Ideas for Balance Conditions?
Potential Balance Conditions

- Left and right subtrees of the root have equal number of nodes (Not strong enough!)
  (e.g., height mismatch)

- Left and right subtrees of the root also have equal height
  (still not enough!)
  (e.g., double chain)
Potential Balance Conditions

• Left and right subtrees of every node have equal # nodes.
  (Too strong! (only perfect trees) $2^h - 1$ nodes)

• Left and right subtrees of every node have equal height.
  (Too strong! (still only perfect trees))
Potential Balance Conditions

Left and right subtrees of every node have heights differing by at most 1

If we define

\[ \text{balance}(\text{node}) = \text{height}(\text{node.left}) - \text{height}(\text{node.right}) \]

Maintain property for every node

\[-1 \leq \text{balance}(\text{node}) \leq 1\]
AVL Tree

A kind of self-balancing binary search tree!
AVL Tree (Bonus material: etymology)

Invented by Georgy Adelson-Velsky and Evgenii Landis in 1962
The **AVL Tree** Data Structure

An **AVL tree** is a *self-balancing* binary search tree.

**Structural properties**

1. **Binary tree** property (same as BST)
2. **Order** property (same as for BST)
3. **Balance condition:**
   - balance of every node is between -1 and 1
   - where $\text{balance}(node) = \text{height}(node.\text{left}) - \text{height}(node.\text{right})$

Result: **Worst-case** depth is $O(\log n)$
Example #1: Is this an AVL Tree?

**Balance Condition:**
balance of every node is between -1 and 1

where $\text{balance}(node) = \text{height}(node\text{.left}) - \text{height}(node\text{.right})$

Yes! Because the left and right subtree of every node have heights differing by at most one
Example #2: Is this an AVL Tree?

**Balance Condition:**
balance of every node is between -1 and 1

where balance\( (node) = \) 
\[ \text{height}(\text{node.left}) - \text{height}(\text{node.right}) \]
AVL Trees

Good News:
Because height of AVL tree is $O(\log(n))$, then find $O(\log(n))$.

But as we insert and delete elements, we need to:
1. Track balance
2. Detect imbalance
3. Restore balance
AVL Trees

Node object
- value: 10
- height: 3
- children:
  - left
  - right

Track height at all times!
AVL tree operations

• **AVL find:**
  • Same as usual BST find

• **AVL insert:**
  - First, usual generic BST insert
  - then check balance condition
  - and potentially “fix” the tree
  - Four different imbalance case

• **AVL delete:**
  • The “easy way” is lazy deletion
  • Otherwise, do the deletion and then check for several imbalance cases (we will skip this)
First insert example

Insert(6)
Insert(3)
Insert(1)

Third insertion

 violates balance condition
(happens to be at the root in this example)

What’s the only way to fix it?
Fix: Apply “Single Rotation”

- **Single rotation**: The basic operation we’ll use to rebalance
  - Move child of unbalanced node into parent position
  - Parent becomes the “other” child (always okay in a BST!)
  - Other subtrees move in only way BST allows (we’ll see in generalized example)
Generalizing our examples...
Generalizing our examples...
Generalizing our examples...
Generalizing our examples...
Generalized Single Rotation

rotation on d

Right Rotation
Generalized Single Rotation

Left Rotation

Diagram showing a tree with nodes labeled A, B, C, D, and E before and after a left rotation.
Single Rotations

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AVL Tree insert (more specific):

1. Insert the new node as in our generic BST (a new leaf)

2. For each node on the path from the root to the new leaf, the insertion may (or may not) have changed the node’s height

3. So after insertion in a subtree, detect height imbalance and perform rotation to restore balance at that node

4. Always look for the deepest node that is unbalanced
Case #1: Left-Left Case

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Example #2 for left-left case: insert(16)
Case #2: Right-Right Case

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Case #2: Left-Left Case

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Example for right-right case: \texttt{insert(26)}
Case #3: \textit{Right - Left Case}

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A Better Look at Case #3:

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Case #3: Right-Left Case (after one rotation)

(Figures by Melissa O’Neill, reprinted with her permission to Lilian)
Case #3: Right-Left Case (after two rotations)

A way to remember it:
Move d to grandparent’s position. Put everything else in their only legal positions for a BST.

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