CSE 373

MAY 31\textsuperscript{TH} – EVAN’S FUN LECTURE
ASSORTED MINUTIAE

- Exam Review – Friday 4:30 – 6:00 EEB 105
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• Section will be exam review
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• Practice exams will be out tonight
  • I will link some other practice finals as well, but they can be found on the 17wi page right now.
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• Exam: Tue Jun 6, 2:30 – 4:20
TODAY’S LECTURE

• Interesting topics for implementation
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  • Randomization Rant
TODAY’S LECTURE

• Interesting topics for implementation
  • Randomization Rant
  • Hardware constraints
RANDOMIZATION

• Guess and check
RANDOMIZATION

- Guess and check
  - How bad is it?
RANDOMIZATION

• Guess and check
  • How bad is it?
    • Necessary for some hard problems
RANDOMIZATION

• Guess and check
  • How bad is it?
    • Necessary for some hard problems
    • Still can be useful for some easier problems
RANDOMIZATION

• If an algorithm has a chance $P$ of returning the correct answer to an NP-complete problem in $O(n^k)$ time
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  • $P$ is our success probability
  • NP-complete means we can check a solution in $O(n^k)$ time, but we can find the exact solution in $O(k^n)$ time – very bad
  • Suppose we want to have a confidence equal to $\alpha$, how do we get this?
RANDOMIZATION

• Even if P is low, we can increase our chance of finding the correct solution by running our randomized estimator multiple times
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  • How many times do we need to run our algorithm to be sure our chance of error is less than $\alpha$?
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RANDOMIZATION

\[(1-p)^k = \alpha\]
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\[k \times \ln(1-p) = \ln \alpha\]

\[k = \frac{(\ln \alpha)}{(\ln(1-p))}\]

\[k = \log_{(1-p)} \alpha\]
RANDOMIZATION

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• Cool, I guess… but what does this mean?
• Suppose $P = 0.5$ (we only have a 50% chance of success on any given run) and $\alpha = 0.001$, we only tolerate a 0.1% error
• How many runs do we need to get this level of confidence?
  • Only 10! This is a constant multiple
RANDOMIZATION

• In fact, suppose we always want our error to be 0.1%, how does this change with p?
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RANDOMIZATION

• Even if p is 0.1, only a 10% chance of success, we only need to run the algorithm 80 times to get a 0.001 confidence level
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• What does this mean?
  • Randomized algorithms don’t have to be complicated, if you can create a reasonable guess and can verify it in a short amount of time, then you can get good performance just from running repeatedly.
MINCUT

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• Why do we even care?
MINCUT

• Suppose there is a graph G(V,E)

• Find the two non-empty subgraphs $V_1$ and $V_2$ such that $V_1 \cup V_2 = V$ and the set of edges connecting them are minimal

• Why do we even care?
  • The min-cut is the maximum flow, if we are trying to connect two cities, the limit of traffic flow between nodes in the network
MAX-FLOW MIN-CUT THEOREM (Ford-Fulkerson, 1956): In any network, the value of the max flow is equal to the value of the min cut.

- "Good characterization."
- Proof IOU.

Cut capacity = 28
Flow value = 28
Algorithm [ edit ]

Let \( G(V, E) \) be a graph, and for each edge from \( u \) to \( v \), let \( c(u, v) \) be the capacity and \( f(u, v) \) be the flow. We want to find the maximum flow from the source \( s \) to the sink \( t \). After every step in the algorithm the following is maintained:

**Capacity constraints:**
\[ \forall (u, v) \in E \quad f(u, v) \leq c(u, v) \]

The flow along an edge can not exceed its capacity.

**Skew symmetry:**
\[ \forall (u, v) \in E \quad f(u, v) = -f(v, u) \]

The net flow from \( u \) to \( v \) must be the opposite of the net flow from \( v \) to \( u \) (see example).

**Flow conservation:**
\[ \forall u \in V : u \neq s \text{ and } u \neq t \implies \sum_{w \in V} f(u, w) = 0 \]

That is, unless \( u \) is \( s \) or \( t \). The net flow to a node is zero, except for the source, which "produces" flow, and the sink, which "consumes" flow.

**Value(\( f \):**
\[ \sum_{(s, u) \in E} f(s, u) = \sum_{(v, t) \in E} f(v, t) \]

That is, the flow leaving from \( s \) must be equal to the flow arriving at \( t \).

This means that the flow through the network is a legal flow after each round in the algorithm. We define the residual network \( G_f(V, E_f) \) to be the network with capacity \( c_f(u, v) = c(u, v) - f(u, v) \) and no flow. Notice that it can happen that a flow from \( v \) to \( u \) is allowed in the residual network, though disallowed in the original network: if \( f(u, v) > 0 \) and \( c(v, u) = 0 \) then \( c_f(v, u) = c(v, u) - f(v, u) = f(u, v) > 0 \).

**Algorithm** Ford–Fulkerson

**Inputs** Given a Network \( G = (V, E) \) with flow capacity \( c \), a source node \( s \), and a sink node \( t \)

**Output** Compute a flow \( f \) from \( s \) to \( t \) of maximum value

1. \( f(u, v) \leftarrow 0 \) for all edges \( (u, v) \)
2. While there is a path \( p \) from \( s \) to \( t \) in \( G_f \), such that \( c_f(u, v) > 0 \) for all edges \( (u, v) \in p \):
   1. Find \( c_f(p) = \min \{ c_f(u, v) : (u, v) \in p \} \)
   2. For each edge \( (u, v) \in p \)
      1. \( f(u, v) \leftarrow f(u, v) + c_f(p) \) (Send flow along the path)
      2. \( f(v, u) \leftarrow f(v, u) - c_f(p) \) (The flow might be "returned" later)

The path in step 2 can be found with for example a breadth-first search or a depth-first search in \( G_f(V, E_f) \). If you use the former, the algorithm is called Edmonds–Karp.
FORD-FULKERSON

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KARGER'S ALGORITHM

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- Contract edges at random!
  - How many edges will you contract to get two subgraphs?
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• Can we estimate the min-cut?
  • What might be an easy estimator?
• Contract edges at random!
  • How many edges will you contract to get two subgraphs?
  • Only $|V|-2$
KARGER'S ALGORITHM

• Does this work?
KARGER'S ALGORITHM

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  • Success probability of $2/|E|$
KARGER'S ALGORITHM

• Does this work?
  • Success probability of $2/|E|$
  • Run it $O(E)$ times, and you have a bounded success rate!
HARDWARE CONSTRAINTS

• So far, we’ve taken for granted that memory access in the computer is constant and easily accessible
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• So far, we’ve taken for granted that memory access in the computer is constant and easily accessible
  • This isn’t always true!
  • At any given time, some memory might be cheaper and easier to access than others
  • Memory can’t always be accessed easily
  • Sometimes the OS lies, and says an object is “in memory” when it’s actually on the disk
HARDWARE CONSTRAINTS

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  • This isn’t feasible to provide!
  • Sometimes there isn’t enough available, and so memory that hasn’t been used in a while gets pushed to the disk
• Memory that is frequently accessed goes to the cache, which is even faster than RAM
The Memory Mountain

Pentium III Xeon
550 MHz
16 KB on-chip L1 d-cache
16 KB on-chip L1 i-cache
512 KB off-chip unified L2 cache

Slopes of Spatial Locality

Ridges of Temporal Locality

read throughput (MB/s)

stride (words)

working set size (bytes)
LOCALITY AND PAGES

• So, the OS does two smart things
  • Spatial locality – if you use memory index Ox371347AB, you are likely to need Ox371347AC – bring both into cache
  • These are called pages, and they are usually around 4kb
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  • Spatial locality – if you use memory index Ox371347AB, you are likely to need Ox371347AC – bring both into cache
  • These are called pages, and they are usually around 4kb
  • All of the processes on your computer have access to pages in memory.
LOCALITY AND PAGES

• When you call new in Java, you are requesting new memory from the heap. If there isn’t memory there, the JVM needs to get new memory from the OS.
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  • The OS only uses memory in page sizes
  • So if you allocate 100Bytes of data, you overallocate to 4kb!
  • But you can use that 4kb if you need more
LOCALITY AND PAGES

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  • Bring recently used data into faster memory
LOCALITY AND PAGES

- Secondly, the OS uses temporal locality,
  - Memory recently accessed is likely to be accessed again
  - Bring recently used data into faster memory
- Types of memory (by speed)
  - Register
  - L1,L2,L3
  - Memory
  - Disk
  - The interwebs (the cloud)
LOCALITY AND PAGES

• The OS is always processing this information and deciding which is the best
  • This is why arrays are faster in practice, they are always next to each other in memory
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  • Each new node in a tree may not even be in the same page in memory!!
LOCALITY AND PAGES

• The OS is always processing this information and deciding which is the best
  • This is why arrays are faster in practice, they are always next to each other in memory
  • Each new node in a tree may not even be in the same page in memory!!
• Important to consider when designing and explaining design problems.
FRIDAY

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