CSE373: Data Structures and Algorithms

Dictionaries and Binary Search Trees

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This lecture material represents the work of multiple instructors at the University of Washington. Thank you to all who have contributed!

Today's Outline

Today's Topics

- Finish Asymptotic Analysis
- Dictionary ADT (a.k.a. Map): associate keys with values
  - Extremely common
- Binary Trees

Summary of Asymptotic Analysis

Analysis can be about:

- The problem or the algorithm (usually algorithm)
- Time or space (usually time)
  - Or power or dollars or …
- Best-, worst-, or average-case (usually worst)
- Upper-, lower-, or tight-bound (usually upper)
- The most common thing we will do is give an O upper bound to the worst-case **running** time of an algorithm.

Big-Oh Caveats

- Asymptotic complexity focuses on behavior for large *n* and is independent of any computer / coding trick
- But you can "abuse" it to be misled about trade-offs
- Example: $n^{1/10}$ vs. `log` *n*
  - Asymptotically $n^{1/10}$ grows more quickly
  - But the "cross-over" point is around 5 * 10^{17}
  - So if you have input size less than 2^{58}, prefer $n^{1/10}$
- For *small n*, an algorithm with worse asymptotic complexity might be faster
  - If you care about performance for small *n* then the constant factors can matter

Addendum: Timing vs. Big-Oh Summary

- Big-oh is an essential part of computer science's mathematical foundation
  - Examine the algorithm itself, not the implementation
  - Reason about (even prove) performance as a function of *n*
- Timing also has its place
  - Compare implementations
  - Focus on data sets you care about (versus worst case)
  - Determine what the constant factors "really are"

Let's take a breath

- So far we've covered
  - Some simple ADTs: stacks, queues, lists
  - Some math (proof by induction)
  - How to analyze algorithms
  - Asymptotic notation (Big-Oh)
- Coming up….
  - Many more ADTs
    - Starting with dictionaries

The Dictionary (a.k.a. Map) ADT

- Data:
  - set of (key, value) pairs
  - keys must be comparable
- Operations:
  - `insert(key,value)`
  - `find(key)`
  - `delete(key)`
  - …

insert(david, ….)

find(megan)

Megan Hopp, …

- david
  David Swanson
  OH: Wed 3.30-4.20
  …
- nicholas
  Nicholas Shahan
  OH: Wed 11.30-12.20
  …
- megan
  Megan Hopp
  OH: Mon 10-10.50
  …

Will tend to emphasize the keys;
don't forget about the stored values

A Modest Few Uses

Any time you want to store information according to some key and be able to retrieve it efficiently
- Lots of programs do that!

- Search: inverted indexes, phone directories, …
- Networks: router tables
- Operating systems: page tables
- Compilers: symbol tables
- Databases: dictionaries with other nice properties
- Biology: genome maps
- …

Possibly the most widely used ADT

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
Unsorted linked-list	$O(1)$*	$O(n)$	$O(n)$
Unsorted array	$O(1)$*	$O(n)$	$O(n)$
Sorted linked list	$O(n)$	$O(n)$	$O(n)$
Sorted array	$O(n)$	$O(\log n)$	$O(n)$

* Unless we need to check for duplicates

We'll see a Binary Search Tree (BST) probably does better
but not in the worst case (unless we keep it balanced)

Lazy Deletion

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making `delete` as fast as `find`:
- Instead of actually removing the item just mark it deleted

Plusses:
- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

Minuses:
- Extra *space* for the "is-it-deleted" flag
- Data structure full of deleted nodes wastes *space*
- May complicate other operations

Better dictionary data structures

There are many good data structures for (large) dictionaries

1. Binary trees
2. AVL trees
   - Binary search trees with *guaranteed balancing*
3. Hashtables
   - Not tree-like at all

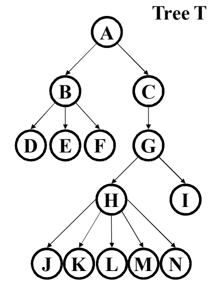
Skipping: Other trees (e.g., B-trees, red-black, splay)

Tree terms (review?)

Root (tree)
Leaves (tree)
Children (node)
Parent (node)
Siblings (node)
Ancestors (node)
Descendents (node)
Subtree (node)

Depth (node)
Height (tree)
Degree (node)
Branching factor (tree)

Tree T



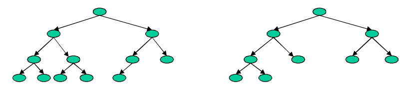
More tree terms

- There are many kinds of trees
  - Every binary tree is a tree
  - Every list is kind of a tree (think of “next” as the one child)
- There are many kinds of binary trees
  - Every binary search tree is a binary tree
  - Later: A binary heap is a different kind of binary tree
- A tree can be balanced or not
  - A balanced tree with *n* nodes has a height of *O*(`log` *n*)
  - Different tree data structures have different “balance conditions” to achieve this

Kinds of trees

Certain terms define trees with specific structure

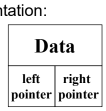
- Binary tree: Each node has at most 2 children (branching factor 2)
- *n*-ary tree: Each node has at most *n* children (branching factor *n*)
- Perfect tree: Each row completely full
- Complete tree: Each row completely full except maybe the bottom row, which is filled from left to right

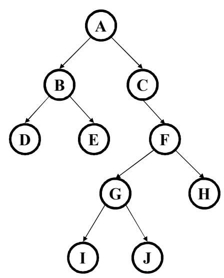


What is the height of a perfect binary tree with n nodes?
A complete binary tree?

Binary Trees

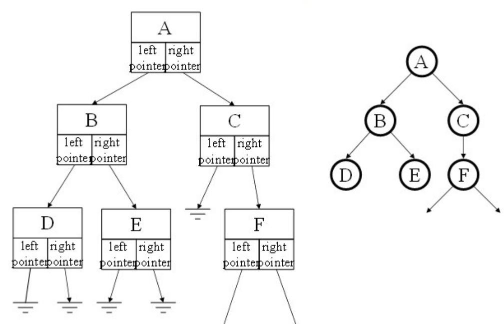
- Binary tree: Each node has at most 2 children (branching factor 2)
- Binary tree is
  - A root *(with data)*
  - A left subtree *(may be empty)*
  - A right subtree *(may be empty)*
- Representation:





- For a dictionary, data will include a key and a value

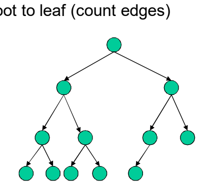
Binary Tree Representation

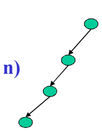


Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:
- max # of leaves: 2^h
- max # of nodes: $2^{(h+1)} - 1$
- min # of leaves: 1
- min # of nodes: $h + 1$





For n nodes, we cannot do better than O(`log` n) height and we want to avoid O(n) height

Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {

              ???

}
```

Calculating height

What is the height of a tree with root `root`?

```
int treeHeight(Node root) {
  if(root == null)
    return -1;
  return 1 + max(treeHeight(root.left),
                 treeHeight(root.right));
}
```

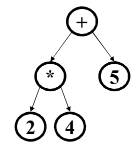
Running time for tree with *n* nodes: *O*(*n*) – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

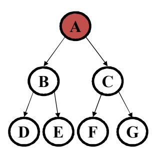
- *Pre-order*: root, left subtree, right subtree
  + * 2 4 5
- *In-order*: left subtree, root, right subtree
  2 * 4 + 5
- *Post-order*: left subtree, right subtree, root
  2 4 * 5 +



(an expression tree)

More on traversals

```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
    process(t.element);
    inOrderTraversal(t.right);
  }
}
```



A = current node A = processing (on the call stack)

A = completed node ✓= element has been processed

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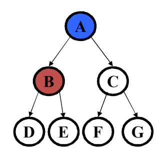

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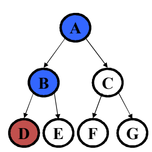

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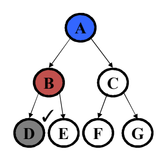

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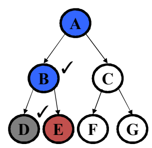

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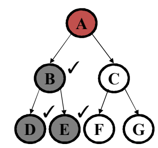

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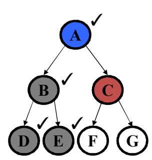

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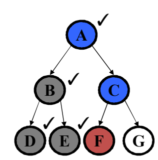

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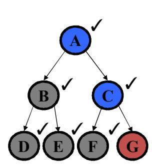

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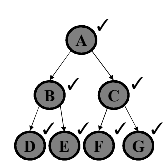

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