CSE373: Data Structures and Algorithms

Math Review

Steve Tanimoto
Winter 2016

This lecture material represents the work of multiple instructors at the University of Washington. Thank you to all who have contributed!

Today

- Review of math essential to algorithm analysis
  - Logarithms and exponents
  - Floor and ceiling functions
- Begin algorithm analysis

Logarithms and Exponents

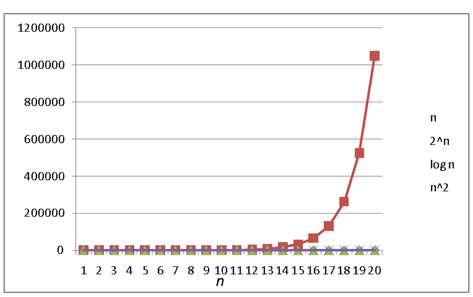
- Definition: `x = 2`^{y} `if log`$_2$ `x = y`
  - 8 = 2^3, so `log`$_2$ 8 = 3
  - 65536= 2^{16}, so `log`$_2$ 65536 = 16
- The exponent of a number says how many times to use the number in a multiplication. e.g. $2^3 = 2 \times 2 \times 2 = 8$
  (2 is used 3 times in a multiplication to get 8)
- A logarithm says how many of one number to multiply to get another number. It asks "what exponent produced this?"
  e.g. `log`$_2$8 = 3 *(2 makes 8 when used 3 times in a multiplication)*

Logarithms and Exponents

- Definition: `x = 2`^{y} `if log`$_2$ `x = y`
  - 8 = 2^3, so `log`$_2$ 8 = 3
  - 65536= 2^{16}, so `log`$_2$ 65536 = 16
- Since so much is binary in CS, `log` almost always means `log`$_2$
- `log`$_2$ n tells you how many bits needed to represent n combinations.
- So, `log`$_2$ 1,000,000 = "a little under 20"
- Logarithms and exponents are inverse functions. Just as exponents grow *very* quickly, logarithms grow *very* slowly.

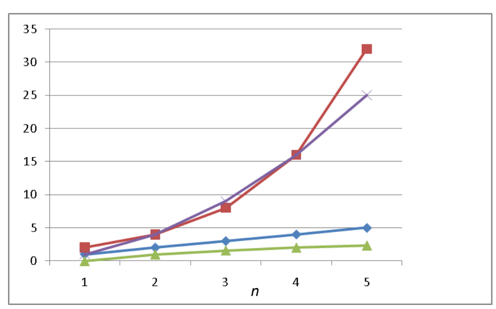
Logarithms and Exponents

See Excel file for plot data – play with it!



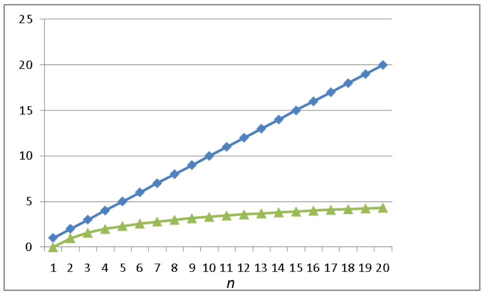
Logarithms and Exponents

See Excel file for plot data – play with it!



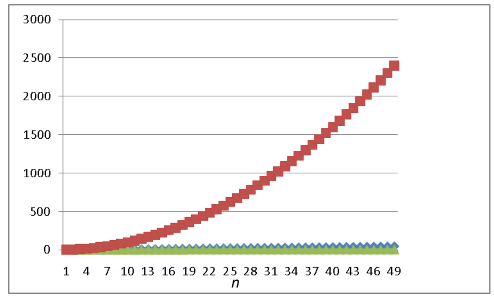
Logarithms and Exponents

See Excel file for plot data – play with it!



Logarithms and Exponents

See Excel file for plot data – play with it!



Properties of logarithms

- $\log(A \cdot B) = \log A + \log B$
- $\log(N^k) = k \log N$
- $\log(A/B) = \log A - \log B$
- $\log(\log x)$ is written $\log \log x$
  - Grows as slowly as 2^{2^y} grows quickly
- $(\log x)(\log x)$ is written $\log^2 x$
  - It is greater than $\log x$ for all $x > 2$
  - It is not the same as $\log \log x$

Log base doesn't matter much!

"Any base *B* log is equivalent to base 2 log within a constant factor"
- And we are about to stop worrying about constant factors!
- In particular, $\log_2 x = 3.22 \log_{10} x$
- In general we can convert log bases via a constant multiplier
- To convert from base B to base A:

  $\log_B x = (\log_A x) / (\log_A B)$

Floor and ceiling

$\lfloor X \rfloor$ Floor function: the largest integer $\leq$ X

$\lfloor 2.7 \rfloor = 2 \qquad \lfloor -2.7 \rfloor = -3 \qquad \lfloor 2 \rfloor = 2$

$\lceil X \rceil$ Ceiling function: the smallest integer $\geq$ X

$\lceil 2.3 \rceil = 3 \qquad \lceil -2.3 \rceil = -2 \qquad \lceil 2 \rceil = 2$

Facts about floor and ceiling

1. $X - 1 < \lfloor X \rfloor \leq X$
2. $X \leq \lceil X \rceil < X + 1$
3. $\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$ if n is an integer