CSE373: Data Structures and Algorithms

Hashing I

Steve Tanimoto
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This lecture material represents the work of multiple instructors at the University of Washington. Thank you to all who have contributed!

Motivating Hash Tables

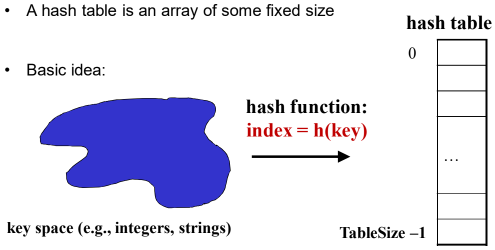
For a **dictionary** with *n* key, value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$	$O(n)$	$O(n)$
• Unsorted array	$O(1)$	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\log n)$	$O(n)$
• *Balanced* tree	$O(\log n)$	$O(\log n)$	$O(\log n)$
• Magic array	$O(1)$	$O(1)$	$O(1)$

Sufficient "magic":
- Use key to compute array index for an item in $O(1)$ time [doable]
- Have a different index for every item [magic]

Hash Tables

- Aim for constant-time (i.e., $O(1)$) **`find`**, **`insert`**, and **`delete`**
  - "On average" under some often-reasonable assumptions
- A hash table is an array of some fixed size
- Basic idea:



Hash Tables vs. Balanced Trees

- In terms of a Dictionary ADT for just **`insert`**, **`find`**, **`delete`**, hash tables and balanced trees are just different data structures
  - Hash tables $O(1)$ on average (*assuming* few *collisions*)
  - Balanced trees $O(\log n)$ worst-case
- Constant-time is better, right?
  - Yes, but you need "hashing to behave" (must avoid collisions)
  - Yes, but **`findMin`**, **`findMax`**, **`predecessor`**, and **`successor`** go from $O(\log n)$ to $O(n)$, **`printSorted`** from $O(n)$ to $O(n \log n)$
  - Hashtables typically implement a different ADT from BSTs. (Keys do not have to be totally ordered.)

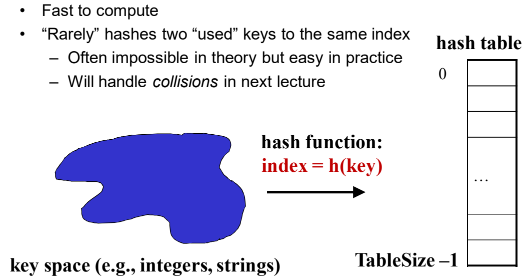
Hash Tables

- There are *m* possible keys (*m* typically large, even infinite)
- We expect our table to have only *n* items
- *n* is much less than *m* (often written $n << m$)

Many dictionaries have this property
- Compiler: All possible identifiers allowed by the language vs. those used in some file of one program
- Database: All possible student names vs. students enrolled
- AI: All possible chess-board configurations vs. those considered by the current player
- …

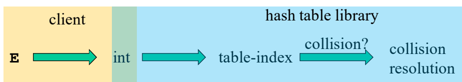
Hash functions

An ideal hash function:
- Fast to compute
- "Rarely" hashes two "used" keys to the same index
  - Often impossible in theory but easy in practice
  - Will handle *collisions* in next lecture



Who hashes what?

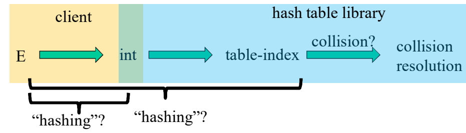
- Hash tables can be generic
  - To store elements of type **E**, we just need **E** to be:
    Hashable: convert any **E** to an **int**
- When hash tables are a reusable library, the division of responsibility generally breaks down into two roles:



- We will learn both roles, but most programmers "in the real world" spend more time as clients while understanding the library

More on roles

Some ambiguity in terminology on which parts are "hashing"



Two roles must both contribute to minimizing collisions (heuristically)

- Client should aim for different ints for expected items
  - Avoid "wasting" any part of **E** or the 32 bits of the **int**
- Library should aim for putting "similar" keys to different locations.
  - Conversion to index is almost always "mod tablesize"
  - Using prime numbers for tablesize is common

What to hash?

We will focus on the two most common things to hash: ints and strings

- For objects with several fields, usually best to have most of the "identifying fields" contribute to the hash to avoid collisions
- Example:

```
class Person {
    String first; String middle; String last;
    Date birthdate;
}
```

- An inherent trade-off: hashing-time vs. collision-avoidance
  - Bad idea(?): Use only first name
  - Good idea(?): Use only middle initial
  - Admittedly, what-to-hash-with is often unprincipled ☹

Hashing integers

- key space = integers
- Simple hash function:
  h(key) = key % TableSize
  - Client: **f(x) = x**
  - Library **g(x) = x % TableSize**
  - Fairly fast and natural
- Example:
  - **TableSize** = 10
  - Insert 7, 18, 41, 34, 10
  - (As usual, ignoring data "along for the ride")

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

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Collision-avoidance

- With "`x % TableSize`" the number of collisions depends on
  – the ints inserted (obviously)
  – `TableSize`
- Larger tablesize tends to help, but not always
  – Example: 70, 24, 56, 43, 10
  with `TableSize` = 10 and `TableSize` = 60
- Technique: Pick tablesize to be prime. Why?
  – Real-life data tends to have a pattern
  – "Multiples of 61" are probably less likely than "multiples of 60"
  – Next lecture shows one collision-handling strategy does *provably* well with prime tablesize

More on prime table size

If `TableSize` is 60 and…
– Lots of data items are multiples of 5, wasting 80% of table
– Lots of data items are multiples of 10, wasting 90% of table
– Lots of data items are multiples of 2, wasting 50% of table

If `TableSize` is 61…
– Collisions can still happen, but 5, 10, 15, 20, … will fill table
– Collisions can still happen but 10, 20, 30, 40, … will fill table
– Collisions can still happen but 2, 4, 6, 8, … will fill table

This "table-filling" property happens whenever the multiple and the tablesize have a *greatest-common-divisor* of 1

Okay, back to the client

- If keys aren't `int`s, the client must convert to an `int`
  – Trade-off: speed versus distinct keys hashing to distinct `int`s
- Very important example: Strings
  – Key space K = $s_0 s_1 s_2 \ldots s_{m-1}$
    - (where s_i are chars: $s_i \in [0,52]$ or $s_i \in [0,256]$ or $s_i \in [0,2^{16}]$)
  – Some choices: Which avoid collisions best?

1. h(K) = s_0 % TableSize
2. h(K) = $\left(\sum_{i=0}^{m-1} s_i \right)$ % TableSize
3. h(K) = $\left(\sum_{i=0}^{k-1} s_i \cdot 37^i \right)$ % TableSize

Specializing hash functions

How might you hash differently if all your strings were web addresses (URLs)?

Combining hash functions

A few rules of thumb / tricks:

1. Use all 32 bits (careful, that includes negative numbers)
2. When smashing two hashes into one hash, use bitwise-xor
   - bitwise-and produces too many 0 bits
   - bitwise-or produces too many 1 bits
3. Rely on expertise of others; consult books and other resources
4. If keys are known ahead of time, try to use a *perfect hash (injective -- no possibility of collisions).*

One expert suggestion

- int result = 17;
- foreach field f
  - int fieldHashcode =
    - boolean: (f ? 1: 0)
    - byte, char, short, int: (int) f
    - long: (int) (f ^ (f >>> 32))
    - float: Float.floatToIntBits(f)
    - double: Double.doubleToLongBits(f), then above
    - Object: object.hashCode( )
  - result = 31 * result + fieldHashcode

Hashing and equality testing

- Need to emphasize a critical detail:
  - We initially *hash* key **`E`** to get a table index
  - To check an item is what we are looking for use its equals method. *equals(E).*
    - Does it have an equal key?
- So a hashtable needs a hash function and a comparator
  - The Java library uses a more object-oriented approach: each object has methods **`equals`** and **`hashCode`**

```
class Object {
  boolean equals(Object o) {…}
  int hashCode() {…}
  …
}
```


Equal Objects Must Hash the Same

- The Java library makes a crucial assumption clients must satisfy
  - And all hash tables make analogous assumptions
- Object-oriented way of saying it:
  If **`a.equals(b)`**, then **`a.hashCode()==b.hashCode()`**
- Why is this essential?
- Why is this up to the client?
- So *always* override **`hashCode`** *correctly* if you override **`equals`**
  - Many libraries use hash tables on your objects

Example

```
class MyDate {
  int month;
  int year;
  int day;

  boolean equals(Object otherObject) {
     if(this==otherObject) return true; // common?
     if(otherObject==null) return false;
     if(getClass()!=other.getClass()) return false;
     return month = otherObject.month
            && year = otherObject.year
            && day = otherObject.day;
  }
}
```

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     return month = otherObject.month
            && year = otherObject.year
            && day = otherObject.day;
  }
  // wrong: must also override hashCode!
}
```


Tougher example

- Suppose you had a **`Fraction`** class where **`equals`** returned **`true`** for 1/2 and 3/6, etc.
- Then must override **`hashCode`** and cannot hash just based on the numerator and denominator
  - Need 1/2 and 3/6 to hash to the same int
- If you write software for a living, you are less likely to implement hash tables from scratch than you are likely to encounter this issue

Conclusions and notes on hashing

- The hash table is one of the most important data structures
  - Supports only **`find`**, **`insert`**, and **`delete`** efficiently
  - Have to search entire table for other operations
- Important to use a good hash function
- Important to keep hash table at a good size
- Side-comment: hash functions have uses beyond hash tables
  - Examples: Cryptography, check-sums
- Big remaining topic: Handling collisions