Announcements
Other Data Structures and Algorithms

- **Quadtrees**: used in spatial applications like geography and image processing
- **Octrees**: used in vision and graphics
- **Image pyramids**: used in image processing and computer vision
- **Backtracking search**: used in AI and vision
- **Graph matching**: used in AI and vision
Quadtrees

- Finkel and Bentley, 1974
- Lots of work by Hanan Samet, including a book
- Raster structure: divides space, not objects
- Form of *block coding*: compact storage of a large 2-dimensional array
- Vector versions exist too
Quadtrees, the idea

1, 4, 16, 64, 256 nodes
Quadtrees, the idea

Choropleth raster map
Quadtrees

- Grid with $2^k$ times $2^k$ pixels
- Depth is $k + 1$
- Internal nodes always have 4 children
- Internal nodes represent a non-homogeneous region
- Leaves represent a homogeneous region and store the common value (or name)
Quadtree complexity theorem

• A subdivision with boundary length $r$ pixels in a grid of $2^k$ times $2^k$ gives a quadtree with $O(k \cdot r)$ nodes.

• Idea: two adjacent, different pixels “cost” at most 2 paths in the quadtree.
Overlay with quadtrees

Water

Acid rain with PH below 4.5
Result of overlay
Various queries

- Point location: trivial
- Windowing: descend into subtree(s) that intersect query window
- Traversal boundary polygon: up and down in the quadtree
Octrees

- Like quadtrees, but for 3D applications.
- Breaks 3D space into octants
- Useful in graphics for representing 3D objects at different resolutions
Hierarchical space carving

- Big cubes => fast, poor results
- Small cubes => slow, more accurate results
- Combination = octrees

RULES:
- cube's out => done
- cube's in => done
- else => recurse
The rest of the chair
Same for a husky pup
Optimizing the dog mesh

Registered points

Initial mesh

Optimized mesh
Our viewer
Image Pyramids

- Bottom level is the original image.

- 2nd level is derived from the original image according to some function.

- 3rd level is derived from the 2nd level according to the same function.

And so on.

Bottom level is the original image.
Mean Pyramid

Bottom level is the original image.

At 2\textsuperscript{nd} level, each pixel is the mean of 4 pixels in the original image.

At 3\textsuperscript{rd} level, each pixel is the mean of 4 pixels in the 2\textsuperscript{nd} level.

Bottom level is the original image.

And so on.
Gaussian Pyramid
At each level, image is smoothed and reduced in size.

And so on.

At 2nd level, each pixel is the result of applying a Gaussian mask to the first level and then subsampling to reduce the size.

Bottom level is the original image.
Example: Subsampling with Gaussian pre-filtering
Backtracking Search in AI/Vision

- Start at the root of a search tree at a “state”
- Generate children of that state
- For each child
  - If the child is the goal, done
  - If the child does not satisfy the constraints of the problem, ignore it and keep going in this loop
  - Else call the search recursively for this child
- Return

This is called backtracking, because if it goes through all children of a node and finds no solution, it returns to the parent and continues with the children of that parent.
Graph Matching

Input: 2 digraphs $G_1 = (V_1, E_1), G_2 = (V_2, E_2)$

Questions to ask:

1. Are $G_1$ and $G_2$ isomorphic?

2. Is $G_1$ isomorphic to a subgraph of $G_2$?

3. How similar is $G_1$ to $G_2$?

4. How similar is $G_1$ to the most similar subgraph of $G_2$?
Isomorphism for Digraphs

G1 is isomorphic to G2 if there is a 1-1, onto mapping \( h: V_1 \rightarrow V_2 \) such that \( (v_i,v_j) \in E_1 \) iff \( (h(v_i), h(v_j)) \in E_2 \).

Find an isomorphism \( h: \{1,2,3,4,5\} \rightarrow \{a,b,c,d,e\} \).
Check that the condition holds for every edge.

Answer: \( h(1)=b, h(2)=e, h(3)=c, h(4)=a, h(5)=d \)
Isomorphism for Digraphs

\( G_1 \) is isomorphic to \( G_2 \) if there is a 1-1, onto mapping \( h: V_1 \rightarrow V_2 \) such that \((v_i, v_j) \in E_1 \) iff \((h(v_i), h(v_j)) \in E_2\)

Answer: \( h(1) = b, h(2) = e, h(3) = c, h(4) = a, h(5) = d \)

\((1,2) \in E_1 \) and \((h(1), h(2)) = (b, e) \in E_2\).

\((2,1) \in E_1 \) and \((e, b) \in E_2\).

\((2,5) \in E_1 \) and \((e, d) \in E_2\).

\((3,1) \in E_1 \) and \((c, b) \in E_2\).

\((3,2) \in E_1 \) and \((c, e) \in E_2\).

...
Subgraph Isomorphism for Digraphs

G1 is isomorphic to a subgraph of G2 if there is a 1-1 mapping \( h: V1 \rightarrow V2 \) such that \( (v_i,v_j) \in E1 \Rightarrow (h(v_i), h(v_j)) \in E2 \).

Isomorphism and subgraph isomorphism are defined similarly for undirected graphs.

In this case, when \( (v_i,v_j) \in E1 \), either \( (v_i,v_j) \) or \( (v_j,v_i) \) can be listed in \( E2 \), since they are equivalent and both mean \( \{v_i,v_j\} \).
Subgraph Isomorphism for Graphs

G1 is isomorphic to a subgraph of G2 if there is a 1-1 mapping $h: V1 \rightarrow V2$ such that $\{v_i, v_j\} \in E1 \Rightarrow \{h(v_i), h(v_j)\} \in E2$.

Because there are no directed edges, there are more possible mappings:

1 2 3
0 0 0
c b d
c d b (shown on graph)
b c d
b d c
d b c
d c b
Graph Matching Algorithms: Subgraph Isomorphism for Digraph

Given model graph \( M = (VM, EM) \)

data graph \( D = (VD, ED) \)

Find 1-1 mapping \( h: VM \rightarrow VD \)

satisfying \( (vi, vj) \in EM \Rightarrow ((h(vi), h(vj)) \in ED. \)
Method: Recursive Backtracking Tree Search
(Order is depth first, leftmost child first.)

(1,2) ∈ M, but (a,b) ∈ D

YES!
Application to Computer Vision

Find the house model in the image graph.
More Examples
RIO: Relational Indexing for Object Recognition

- RIO worked with industrial parts that could have
  - planar surfaces
  - cylindrical surfaces
  - threads
Object Representation in RIO

- 3D objects are represented by a 3D mesh and set of 2D view classes.

- Each view class is represented by an attributed graph whose nodes are features and whose attributed edges are relationships.

- Graph matching is done through an indexing method, not covered here.
**RIO Features**

- **Ellipses**
- **Coaxials**
- **Coaxials-multi**
- **Parallel lines**
- **Junctions**
- **Triples**
RIO Relationships

• share one arc
• **share one line**
• share two lines
• coaxial
• close at extremal points
• bounding box encloses / enclosed by
Graph Representation

1 coaxials-
multi

encloses

encloses

encloses

3 parallel
lines

2 ellipse

coaxial

This is just a piece of the whole graph.
Sample Alignments
3D to 2D Perspective Projection
Fergus Object Recognition by Parts:

- Enable Computers to Recognize Different Categories of Objects in Images.
Model: Constellation Of Parts

Fischler & Elschlager, 1973
Motorbikes

Part 1 – Det: 5e-18

Part 2 – Det: 8e-22

Part 3 – Det: 6e-18

Part 4 – Det: 1e-19

Part 5 – Det: 3e-17

Part 6 – Det: 4e-24

Background – Det: 5e-19