CSE373: Data Structures & Algorithms
Lecture 24: Parallel Reductions, Maps, and Algorithm Analysis

Lauren Milne
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This week....

• Homework 6 due today!
  – Done with all homeworks 😊

• Course Evaluations – Time at the end of lecture

• Final Exam Friday
  – Final exam review tonight at 7pm
Outline

Done:
• How to write a parallel algorithm with fork and join
• Why using divide-and-conquer with lots of small tasks is best
  – Combines results in parallel
  – (Assuming library can handle “lots of small threads”)

Now:
• More examples of simple parallel programs that fit the “map” or “reduce” patterns
• Teaser: Beyond maps and reductions
• Asymptotic analysis for fork-join parallelism
• Amdahl’s Law
What else looks like this?

• Saw summing an array went from $O(n)$ sequential to $O(\log n)$ parallel (assuming a lot of processors and very large $n$)
  – Exponential speed-up in theory ($n / \log n$ grows exponentially)

• Anything that can use results from two halves and merge them in $O(1)$ time has the same property...
Examples

• Maximum or minimum element
• Is there an element satisfying some property (e.g., is there a 17)?
• Left-most element satisfying some property (e.g., first 17)
• Counts, for example, number of strings that start with a vowel
Reductions

- Computations of this form are called reductions.

![Diagram](image)

- Produce single answer from collection via an associative operator:
  - Associative operator: \( a \ast (b \ast c) = (a \ast b) \ast c \)
  - Examples: max, sum, product, count ...
    - max: \( \text{max}(a, \text{max}(b, c)) = \text{max} \left( \text{max}(a, b), c \right) \)
    - sum: \( a + (b+c) = (a+b) + c \)
    - product: \( a \ast (b \ast c) = (a \ast b) \ast c \)
  - Non-examples: subtraction, exponentiation, median, ...
    - subtraction: \( 5 - (3-2) \neq (5-3) - 2 \)
Even easier: Maps (Data Parallelism)

- A map operates on each element of a collection independently to create a new collection of the same size.

  ![Diagram of map operation](image)

- Canonical example: Vector addition

  ```java
  int[] vector_add(int[] arr1, int[] arr2) {
    assert (arr1.length == arr2.length);
    result = new int[arr1.length];
    FORALL (i=0; i < arr1.length; i++) {
      result[i] = arr1[i] + arr2[i];
    }
    return result;
  }
  ```

<table>
<thead>
<tr>
<th>input</th>
<th>6</th>
<th>4</th>
<th>16</th>
<th>10</th>
<th>16</th>
<th>14</th>
<th>2</th>
<th>8</th>
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</thead>
<tbody>
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<td>10</td>
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<tr>
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<td>16</td>
<td>18</td>
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<td>10</td>
<td>15</td>
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Maps and reductions

Maps and reductions: the “workhorses” of parallel programming

– By far the two most important and common patterns

– Learn to recognize when an algorithm can be written in terms of maps and reductions

– Use maps and reductions to describe (parallel) algorithms

– Programming them becomes “trivial” with a little practice
  • Exactly like sequential for-loops seem second-nature
Beyond maps and reductions

- Some problems are “inherently sequential”
  “Six ovens can’t bake a pie in 10 minutes instead of an hour”

- But not all parallelizable problems are maps and reductions

- If had one more lecture, would show “parallel prefix”, a clever algorithm to parallelize the problem that this sequential code solves

<table>
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<th>14</th>
<th>2</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
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<td>10</td>
<td>26</td>
<td>36</td>
<td>52</td>
<td>66</td>
<td>68</td>
<td>76</td>
</tr>
</tbody>
</table>

```java
int[] prefix_sum(int[] input) {
    int[] output = new int[input.length];
    output[0] = input[0];
    for (int i = 1; i < input.length; i++)
        output[i] = output[i-1] + input[i];
    return output;
}
```
Digression: MapReduce on clusters

- You may have heard of Google’s “map/reduce”
  - Or the open-source version Hadoop

- Idea: Perform maps/reduces on data using many machines
  - The system takes care of distributing the data and managing fault tolerance
  - You just write code to map one element and reduce elements to a combined result

- Separates how to do recursive divide-and-conquer from what computation to perform
  - Separating concerns is good software engineering
Analyzing algorithms

• Like all algorithms, parallel algorithms should be:
  – Correct and Efficient

• For our algorithms so far, correctness is “obvious” so we’ll focus on efficiency
  – Want asymptotic bounds
  – Want to analyze the algorithm without regard to a specific number of processors
  – Here: Identify the “best we can do” if the underlying thread-scheduler does its part
Work and Span

Let $T_P$ be the running time if there are $P$ processors available.

Two key measures of run-time:

- **Work**: How long it would take 1 processor = $T_1$
  - Just “sequentialize” the recursive forking

- **Span**: How long it would take infinite processors = $T_\infty$
  - The longest dependence-chain
  - Example: $O(\log n)$ for summing an array
    - Notice having $> n/2$ processors is no additional help
Our simple examples

- Picture showing all the “stuff that happens” during a reduction or a map: it’s a (conceptual!) DAG
Connecting to performance

- Recall: $T_P = \text{running time if there are } P \text{ processors available}$

- Work = $T_1 = \text{sum of run-time of all nodes in the DAG}$
  - That lonely processor does everything
  - Any topological sort is a legal execution
  - $O(n)$ for maps and reductions

- Span = $T_\infty = \text{sum of run-time of all nodes on the most-expensive path in the DAG}$
  - Note: costs are on the nodes not the edges
  - Our infinite army can do everything that is ready to be done, but still has to wait for earlier results
  - $O(\log n)$ for simple maps and reductions
Speed-up

*Parallelizing algorithms is about decreasing span without increasing work too much*

- Speed-up on $P$ processors: $T_1 / T_P$

- Parallelism is the maximum possible speed-up: $T_1 / T_\infty$
  - At some point, adding processors won’t help
  - What that point is depends on the span

- In practice we have $P$ processors. How well can we do?
  - We cannot do better than $O(T_\infty)$ (“must obey the span”)
  - We cannot do better than $O(T_1 / P)$ (“must do all the work”)

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Examples

\[ T_P = O(\max((T_1 / P), T_∞)) \]

- In the algorithms seen so far (e.g., sum an array):
  - \( T_1 = O(n) \)
  - \( T_∞ = O(\log n) \)
  - So expect (ignoring overheads): \( T_P = O(\max(n/P, \log n)) \)

- Suppose instead:
  - \( T_1 = O(n^2) \)
  - \( T_∞ = O(n) \)
  - So expect (ignoring overheads): \( T_P = O(\max(n^2/P, n)) \)
Amdahl’s Law (mostly bad news)

- So far: analyze parallel programs in terms of work and span
- In practice, typically have parts of programs that parallelize well…
  - Such as maps/reductions over arrays

…and parts that don’t parallelize at all

- Such as reading a linked list, getting input, doing computations where each needs the previous step, etc.
Amdahl’s Law (mostly bad news)

Let the work (time to run on 1 processor) be 1 unit time

Let $S$ be the portion of the execution that can’t be parallelized

Then: $T_1 = S + (1-S) = 1$

Suppose parallel portion parallelizes perfectly (generous assumption)

Then: $T_P = S + (1-S)/P$

So the overall speedup with $P$ processors is (Amdahl’s Law):

$T_1 / T_P = 1 / (S + (1-S)/P)$

And the parallelism (infinite processors) is:

$T_1 / T_\infty = 1 / S$
Why such bad news

\[ \frac{T_1}{T_P} = \frac{1}{S + (1-S)/P} \quad \quad \frac{T_1}{T_\infty} = \frac{1}{S} \]

- Suppose 33% of a program’s execution is sequential
  - Then a billion processors won’t give a speedup over 3

- From 1980-2005, 12 years was long enough to get 100x speedup
  - Now suppose in 12 years, clock speed is the same but you get 256 processors instead of 1
  - For 256 processors to get at least 100x speedup, we need
    \[ 100 \leq 1 \div (S + (1-S)/256) \]
    Which means \( S \leq 0.0061 \) (i.e., 99.4% perfectly parallelizable)
All is not lost

Amdahl’s Law is a bummer!
  – Unparallelized parts become a bottleneck very quickly
  – But it doesn’t mean additional processors are worthless

• We can find new parallel algorithms
  – Some things that seem sequential are actually parallelizable

• We can change the problem or do new things
  – Example: computer graphics use tons of parallel processors
    • Graphics Processing Units (GPUs) are massively parallel!
Moore and Amdahl

- Moore’s “Law” is an observation about the progress of the semiconductor industry
  - Transistor density doubles roughly every 18 months
- Amdahl’s Law is a mathematical theorem
  - Diminishing returns of adding more processors
- Both are incredibly important in designing computer systems
Course evals....

• PLEASE do them
  – I’m giving you time now 😊

• What you liked, what you didn’t like