CSE 373
Winter 2015

Midterm Review

1. Big-Oh

a. Complexity of a find() in a union-find data structure containing N elements (no path compression). (worst case)

O(N)

b. Complexity of push() onto a stack containing N elements implemented with a linked list. (worst case)

O(1)

c. $f(n) = N (\log N)^2 + N^2 \log N$

$O(N^2 \log N)$

d. Complexity of a preorder traversal of a Binary Search Tree containing N elements (worst case)

O(N)

e. Complexity of an IncreaseKey(k, v) on a binary min heap containing N elements. Assume you have a reference to the key k. v is the amount that k should be increased. (worst case)

O(log N)

f. $f(n) = N! + 2^N$

O(N!)

g.
```
int example (int n) {
        int sum = 0;
        for (int i = 0; i < n*n; i++) {
                for (int j = i; j > 0; j=j/2) {
                        sum++;
                }
        }
        return sum;
}
```

$O(N^2 \log N)$

2. Proving Big-Oh (c, and n0)

Suppose $f(n) = 12n^2 + 42n - 3$. Prove that f(n) is $O(n^2)$ using the definition f(n) is O(g(n)) if there exists constants c and n0 such that f(n) <= g(n) for every n >= n0.

Let **n0 = 1,**	$12n^2 + 42n - 3$	$<= c*n^2$
	$12(1) + 42(1) - 3$	$<= c(1)$
	51	**<= c**

3. Best Data Structure

For each of the following tasks, what is the most efficient structure and the worst-case time complexity for performing the operations on the structure. You may choose from the following: **sorted array, sorted linked list, binary search tree, AVL tree, min heap, up tree, stack implemented with a linked list, queue implemented with an array**.

a. Keeping track of next customer at a sandwich shop, with new orders constantly coming in.

Queue, O(N)

b. Finding the next patient to be examined in an emergency room.

Min Heap, O(1)

c. Finding and removing the minimum value.

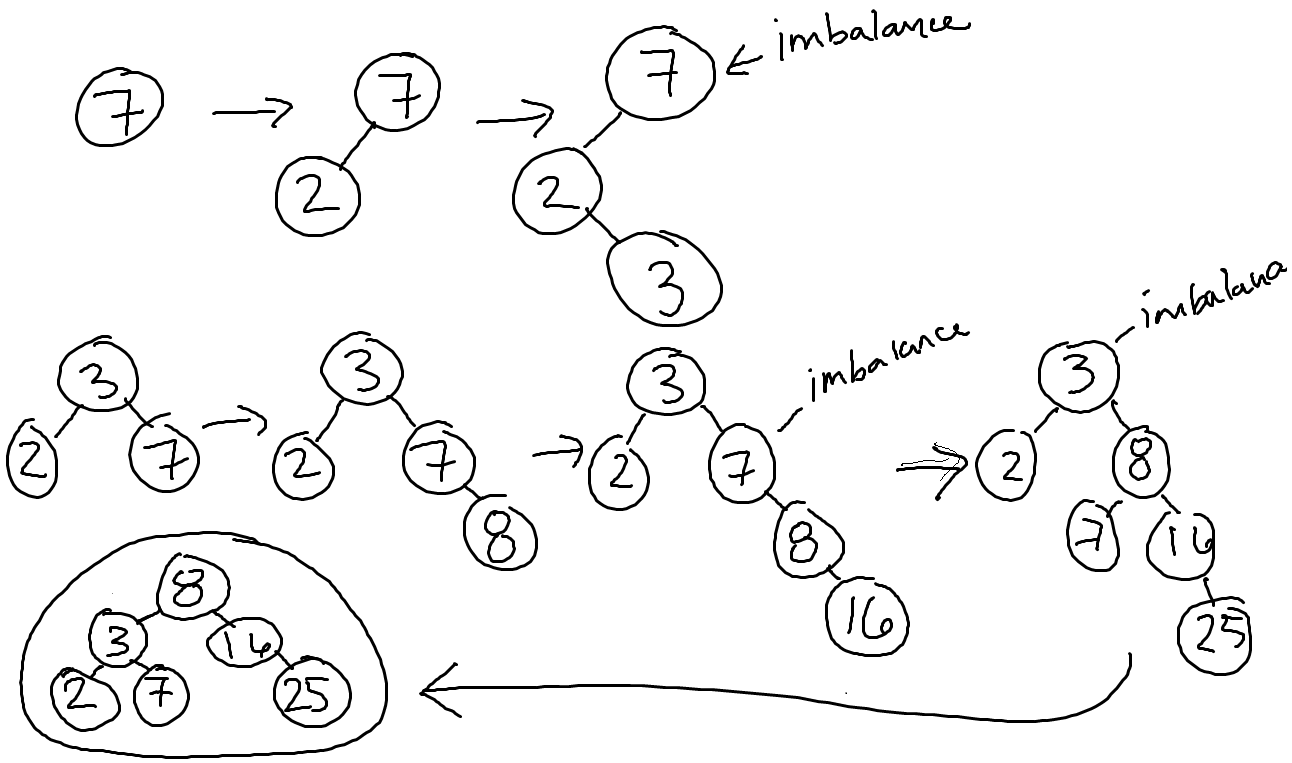
Sorted Linked List, O(1)

d. Inserting a new value.

Stack, O(1)

4. AVL insertion

Insert the following values into an AVL tree (starting with an empty tree). 7, 2, 3, 8, 16, 25. **<u>Circle your final answer if intermediate steps are shown.</u>**



5. **Number of Nodes in Various Types of Trees**

a. What is the minimum and maximum number of nodes in a **binary search tree of height 6**? (Hint: the height of a tree consisting of a single node is 0) Give an exact number.

Minimum = **7**
Maximum = **127**

b. What is the minimum and maximum number of nodes in a **complete binary tree of height 5**?

Minimum = **32**
Maximum = **63**

6. **Proof**

Prove by induction that the *sum of numbers from 0 to N* is equal to *N(N + 1)/2*

0 + 1 + 2 + 3 + 4 + … + N = N(N+1)/2

Base Case: N = 0
0 = (0)(0 + 1)/2
0 = 0

Induction Hypothesis: N = k
Assume the formula to be true for all k, 0 <= k <= N

Induction Step: N = (k + 1)
Sum(0 to k+1) = 0 + 1 + 2 + 3 + … + k + (k+1)
= Sum(0 to k) + (k+1)
= k(k+1)/2 + (k+1) [Induction Hypothesis]
= k(k+1)/2 + 2(k+1)/2
=(k+1)*(k+2)/2
=(k+1)*((k+1)+1)/2

Proven.