CSE 373
Winter 2015

Midterm Review

1. **Big-Oh**
   a. Complexity of a find() in a union-find data structure containing N elements (no path compression). (worst case)

   b. Complexity of push() onto a stack containing N elements implemented with a linked list. (worst case)

   c. $f(n) = N (\log N)^2 + N^2 \log N$

   d. Complexity of a preorder traversal of a Binary Search Tree containing N elements (worst case)

   e. Complexity of an IncreaseKey(k, v) on a binary min heap containing N elements. Assume you have a reference to the key k. v is the amount that k should be increased. (worst case)

   f. $f(n) = N! + 2^N$

   g.
```
int example (int n) {
      int sum = 0;
      for (int i = 0; i < n*n; i++) {
            for (int j = i; j > 0; j=j/2) {
                  sum++;
            }
      }
      return sum;
}
```

2. **Proving Big-Oh (c, and n0)**

Suppose $f(n) = 12n^2 + 42n - 3$. Prove that $f(n)$ is $O(n^2)$ using the definition $f(n)$ is $O(g(n))$ if there exists constants c and n0 such that $f(n) <= g(n)$ for every $n >= n0$.

3. Best Data Structure

For each of the following tasks, what is the most efficient structure and the worst-case time complexity for performing the operations on the structure. You may choose from the following: **sorted array, sorted linked list, binary search tree, AVL tree, min heap, up tree, stack implemented with a linked list, queue implemented with an array**.

a. Keeping track of next customer at a sandwich shop, with new orders constantly coming in.

b. Finding the next patient to be examined in an emergency room.

c. Finding and removing the minimum value.

d. Inserting a new value.

4. AVL insertion

Insert the following values into an AVL tree (starting with an empty tree). 7, 2, 3, 8, 16, 25. **Circle your final answer if intermediate steps are shown.**

5. **Number of Nodes in Various Types of Trees**

a. What is the minimum and maximum number of nodes in a **binary search tree of height 6**? (Hint: the height of a tree consisting of a single node is 0) Give an exact number.

Minimum =
Maximum =

b. What is the minimum and maximum number of nodes in a **complete binary tree of height 5**?

Minimum =
Maximum =

6. **Proof**

Prove by induction that the *sum of numbers from 0 to N* is equal to $N(N + 1)/2$