Admin

- Homework 5 partner selection due TODAY!
  - Catalyst link posted on the webpage

- Homework 5 due next Wednesday at 11pm!
  - Good job for already starting….
The comparison sorting problem

Assume we have $n$ comparable elements in an array and we want to rearrange them to be in increasing order.

Input:
- An array $A$ of data records
- A key value in each data record
- A comparison function (consistent and total)

Effect:
- Reorganize the elements of $A$ such that for any $i$ and $j$, if $i < j$ then $A[i] \leq A[j]$.
- (Also, $A$ must have exactly the same data it started with)
- Could also sort in reverse order, of course.

An algorithm doing this is a comparison sort.
Surprising amount of neat stuff to say about sorting:

Simple algorithms: $O(n^2)$
- Insertion sort
- Selection sort
- Shell sort
- ...

Fancier algorithms: $O(n \log n)$
- Heap sort
- Merge sort
- Quick sort
- ...

Comparison lower bound: $\Omega(n \log n)$

Specialized algorithms: $O(n)$
- Bucket sort
- Radix sort

Handling huge data sets
- External sorting
Divide and conquer

Very important technique in algorithm design

1. Divide problem into smaller parts

2. Independently solve the simpler parts
   - Think recursion
   - Or potential parallelism

3. Combine solution of parts to produce overall solution
Divide-and-Conquer Sorting

Two great sorting methods are fundamentally divide-and-conquer

1. **Merge sort:**
   - Sort the left half of the elements (recursively)
   - Sort the right half of the elements (recursively)
   - Merge the two sorted halves into a sorted whole

2. **Quick sort:**
   - Pick a “pivot” element
   - Divide elements into less-than pivot and greater-than pivot
   - Sort the two divisions (recursively on each)
   - Answer is sorted-less-than then pivot then sorted-greater-than
Quick sort

- A divide-and-conquer algorithm
  - Recursively chop into two pieces
  - Instead of doing all the work as we merge together, we will do all the work as we recursively split into halves
  - Unlike merge sort, does not need auxiliary space

- $O(n \log n)$ on average 😊, but $O(n^2)$ worst-case ☹️

- Faster than merge sort in practice?
  - Often believed so
  - Does fewer copies and more comparisons, so it depends on the relative cost of these two operations!
Quicksort Overview

1. Pick a pivot element

2. Partition all the data into:
   - A. The elements less than the pivot
   - B. The pivot
   - C. The elements greater than the pivot

3. Recursively sort A and C

4. The answer is, “as simple as A, B, C”
Think in Terms of Sets

Select pivot value

Partition S

Quicksort(S₁) and Quicksort(S₂)

Presto! S is sorted

[Weiss]
Example, Showing Recursion

Divide

Divide

Divide

1 Element

Conquer

Conquer

Conquer

Conquer
Details

Have not yet explained:

- How to pick the pivot element
  - Any choice is correct: data will end up sorted
  - But as analysis will show, want the two partitions to be about equal in size

- How to implement partitioning
  - In linear time
  - In place
Pivots

• Best pivot?
  – Median
  – Halve each time

• Worst pivot?
  – Greatest/least element
  – Problem of size n - 1
  – $O(n^2)$
Potential pivot rules

While sorting \texttt{arr} from \texttt{lo} to \texttt{hi-1} …

• Pick \texttt{arr[lo]} or \texttt{arr[hi-1]}
  – Fast, but worst-case occurs with mostly sorted input

• Pick random element in the range
  – Does as well as any technique, but (pseudo)random number
    generation can be slow
  – Still probably the most elegant approach

• Median of 3, e.g., \texttt{arr[lo]}, \texttt{arr[hi-1]}, \texttt{arr[(hi+lo)/2]}
  – Common heuristic that tends to work well
Partitioning

- Conceptually simple, but hardest part to code up correctly
  - After picking pivot, need to partition in linear time in place

- One approach (there are slightly fancier ones):
  1. Swap pivot with arr[lo]
  2. Use two fingers i and j, starting at lo+1 and hi-1
  3. while (i < j)
      - if (arr[j] > pivot) j--
      - else if (arr[i] < pivot) i++
      - else swap arr[i] with arr[j]
  4. Swap pivot with arr[i] *

*skip step 4 if pivot ends up being least element
Example

• Step one: pick pivot as median of 3
  – \( lo = 0, hi = 10 \)

```
  0 1 2 3 4 5 6 7 8 9
  8 1 4 9 0 3 5 2 7 6
```

• Step two: move pivot to the \( lo \) position

```
  0 1 2 3 4 5 6 7 8 9
  6 1 4 9 0 3 5 2 7 8
```
Example

Now partition in place

Move fingers

Swap

Move fingers

Move pivot

Often have more than one swap during partition – this is a short example
Quick sort visualization

• http://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html
Analysis

• Best-case: Pivot is always the median
  \[ T(0) = T(1) = 1 \]
  \[ T(n) = 2T(n/2) + n \quad \text{-- linear-time partition} \]
  Same recurrence as merge sort: \( O(n \log n) \)

• Worst-case: Pivot is always smallest or largest element
  \[ T(0) = T(1) = 1 \]
  \[ T(n) = 1T(n-1) + n \]
  Basically same recurrence as selection sort: \( O(n^2) \)

• Average-case (e.g., with random pivot)
  – \( O(n \log n) \), not responsible for proof (in text)
Cutoffs

• For small $n$, all that recursion tends to cost more than doing a quadratic sort
  – Remember asymptotic complexity is for large $n$

• Common engineering technique: switch algorithm below a cutoff
  – Reasonable rule of thumb: use insertion sort for $n < 10$

• Notes:
  – Could also use a cutoff for merge sort
  – Cutoffs are also the norm with parallel algorithms
    • Switch to sequential algorithm
  – None of this affects asymptotic complexity
Cutoff pseudocode

```java
void quicksort(int[] arr, int lo, int hi) {
    if (hi - lo < CUTOFF)
        insertionSort(arr, lo, hi);
    else
        ...
}
```

Notice how this cuts out the vast majority of the recursive calls
- Think of the recursive calls to quicksort as a tree
- Trims out the bottom layers of the tree
How Fast Can We Sort?

• Heapsort & mergesort have $O(n \log n)$ worst-case running time.
• Quicksort has $O(n \log n)$ average-case running time.
• These bounds are all tight, actually $\Theta(n \log n)$.

• Comparison sorting in general is $\Omega(n \log n)$.
  – An amazing computer-science result: proves all the clever programming in the world cannot comparison-sort in linear time.
The Big Picture

Surprising amount of juicy computer science: 2-3 lectures…

Simple algorithms: $O(n^2)$
- Insertion sort
- Selection sort
- Shell sort

Fancier algorithms: $O(n \log n)$
- Heap sort
- Merge sort
- Quick sort (avg)

Comparison lower bound: $\Omega(n \log n)$

Specialized algorithms: $O(n)$
- Bucket sort
- Radix sort

Handling huge data sets
- External sorting

How???
- Change the model – assume more than “compare(a,b)”
Bucket Sort (a.k.a. BinSort)

• If all values to be sorted are known to be integers between 1 and $K$ (or any small range):
  – Create an array of size $K$
  – Put each element in its proper bucket (a.k.a. bin)
  – If data is only integers, no need to store more than a count of how times that bucket has been used

• Output result via linear pass through array of buckets

```
count array
1   3
2   1
3   2
4   2
5   3
```

• Example:
  K=5
  input (5,1,3,4,3,2,1,1,5,4,5)
  output: 1,1,1,2,3,3,4,4,5,5,5
Visualization

Analyzing Bucket Sort

- Overall: $O(n+K)$
  - Linear in $n$, but also linear in $K$
  - $\Omega(n \log n)$ lower bound does not apply because this is not a comparison sort

- Good when $K$ is smaller (or not much larger) than $n$
  - We don’t spend time doing comparisons of duplicates

- Bad when $K$ is much larger than $n$
  - Wasted space; wasted time during linear $O(K)$ pass

- For data in addition to integer keys, use list at each bucket
**Bucket Sort with Data**

- Most real lists aren’t just keys; we have data
- Each bucket is a list (say, linked list)
- To add to a bucket, insert in $O(1)$ (at beginning, or keep pointer to last element)

<table>
<thead>
<tr>
<th>count array</th>
<th>Rocky V</th>
<th>Harry Potter</th>
<th>Casablanca</th>
<th>Star Wars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>Rocky V</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>Harry Potter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>Casablanca</td>
<td>Star Wars</td>
<td></td>
</tr>
</tbody>
</table>

- Example: Movie ratings; scale 1-5; 1=bad, 5=excellent
- Input=
  5: Casablanca
  3: Harry Potter movies
  5: Star Wars Original Trilogy
  1: Rocky V

- Result: 1: Rocky V, 3: Harry Potter, 5: Casablanca, 5: Star Wars
- Easy to keep ‘stable’; Casablanca still before Star Wars
Radix sort

• Radix = “the base of a number system”
  – Examples will use 10 because we are used to that
  – In implementations use larger numbers
    • For example, for ASCII strings, might use 128

• Idea:
  – Bucket sort on one digit at a time
    • Number of buckets = radix
    • Starting with least significant digit
    • Keeping sort stable
  – Do one pass per digit
  – Invariant: After $k$ passes (digits), the last $k$ digits are sorted

• Aside: Origins go back to the 1890 U.S. census
Example

Radix = 10

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>721</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>537</td>
<td>478</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>143</td>
<td></td>
<td></td>
<td>67</td>
<td>38</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Input: 478
537
9
721
3
38
143
67

First pass:
bucket sort by ones digit

Order now: 721
3
143
537
67
478
38
9
Example

Radix = 10

Order was: 721 3 143 537 67 478 38 9

Second pass: stable bucket sort by tens digit

Order now: 3 9 721 537 143 67 478 9

Winter 2014  CSE373: Data Structures & Algorithms
Example

Radix = 10

Order was: 3 9 721 537 38 143 67 478

Order now: 3 9 721 537 38 143 67 478

Third pass:

**stable** bucket sort by 100s digit
Visualization

Analysis

Input size: $n$
Number of buckets = Radix: $B$
Number of passes = “Digits”: $P$

Work per pass is 1 bucket sort: $O(B+n)$

Total work is $O(P(B+n))$

Compared to comparison sorts, sometimes a win, but often not

- Example: Strings of English letters up to length 15
  - Run-time proportional to: $15*(52 + n)$
  - This is less than $n \log n$ only if $n > 33,000$
  - Of course, cross-over point depends on constant factors of the implementations
    - And radix sort can have poor locality properties
Sorting massive data

• Need sorting algorithms that minimize disk/tape access time:
  – Quicksort and Heapsort both jump all over the array, leading to expensive random disk accesses
  – Merge sort scans linearly through arrays, leading to (relatively) efficient sequential disk access

• Merge sort is the basis of massive sorting

• Merge sort can leverage multiple disks
External Merge Sort

- Sort 900 MB using 100 MB RAM
  - Read 100 MB of data into memory
  - Sort using conventional method (e.g. quicksort)
  - Write sorted 100MB to temp file
  - Repeat until all data in sorted chunks (900/100 = 9 total)
- Read first 10 MB of each sorted chuck, merge into remaining 10MB
  - writing and reading as necessary
  - Single merge pass instead of $\log n$
  - Additional pass helpful if data much larger than memory
- Parallelism and better hardware can improve performance
- Distribution sorts (similar to bucket sort) are also used
Last Slide on Sorting

- Simple $O(n^2)$ sorts can be fastest for small $n$
  - Selection sort, Insertion sort (latter linear for mostly-sorted)
  - Good for “below a cut-off” to help divide-and-conquer sorts
- $O(n \log n)$ sorts
  - Heap sort, in-place but not stable nor parallelizable
  - Merge sort, not in place but stable and works as external sort
  - Quick sort, in place but not stable and $O(n^2)$ in worst-case
    - Often fastest, but depends on costs of comparisons/copies
- $\Omega (n \log n)$ is worst-case and average lower-bound for sorting by comparisons
- Non-comparison sorts
  - Bucket sort good for small number of possible key values
  - Radix sort uses fewer buckets and more phases
- Best way to sort? It depends!