CSE373: Data Structures & Algorithms

Lecture 6: Priority Queues

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A new ADT: Priority Queue

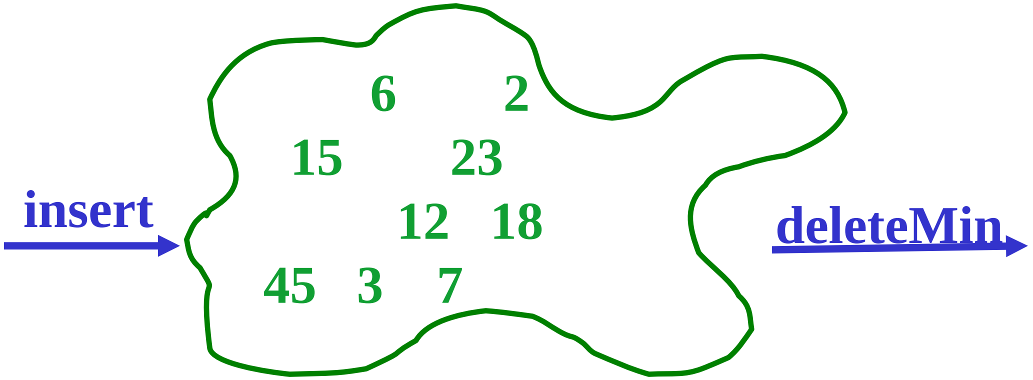
- Textbook Chapter 6
  - Nice to see a new and surprising data structure

- A **priority queue** holds *compare-able data*
  - Like dictionaries and unlike stacks and queues, need to *compare items*
    - Given *x* and *y*, is *x* less than, equal to, or greater than *y*
    - Meaning of the ordering can depend on your data
    - Many data structures require this: dictionaries, sorting
  - Integers are comparable, so will use them in examples
    - But the priority queue ADT is much more general
    - Typically two fields, the *priority* and the *data*

Priorities

- Each item has a "priority"
  - The *lesser* item is the one with the *greater* priority
  - So "priority 1" is more important than "priority 4"
  - (Just a convention, think "first is best")

- Operations:
  - **`insert`**
  - **`deleteMin`**
  - **`is_empty`**



- Key property: **`deleteMin`** *returns* and *deletes* the item with greatest priority (lowest priority value)
  - Can resolve ties arbitrarily

Example

`insert` *x1* with priority *5*

`insert` *x2* with priority *3*

`insert` *x3* with priority *4*

a = **`deleteMin // x2`**

b = **`deleteMin // x3`**

`insert` *x4* with priority *2*

`insert` *x5* with priority *6*

c = **`deleteMin // x4`**

d = **`deleteMin // x1`**

- Analogy: **`insert`** is like **`enqueue`**, **`deleteMin`** is like **`dequeue`**
  - But the whole point is to use priorities instead of FIFO

Applications

Like all good ADTs, the priority queue arises often

- Sometimes blatant, sometimes less obvious

- Run multiple programs in the operating system
  - "critical" before "interactive" before "compute-intensive"
  - Maybe let users set priority level
- Treat hospital patients in order of severity (or triage)
- Select print jobs in order of decreasing length?
- Forward network packets in order of urgency
- Select most frequent symbols for data compression (cf. CSE143)
- Sort (first **`insert`** all, then repeatedly **`deleteMin`**)
  - Much like Homework 1 uses a stack to implement reverse

More applications

- "Greedy" algorithms
  - May see an example when we study graphs in a few weeks

- Discrete event simulation (system simulation, virtual worlds, …)
  - Each event *e* happens at some time *t*, updating system state and generating new events *e1*, …, *en* at times *t*+*t1*, …, *t*+*tn*
  - Naïve approach: advance "clock" by 1 unit at a time and process any events that happen then
  - Better:
    - *Pending events* in a priority queue (priority = event time)
    - Repeatedly: **`deleteMin`** and then **`insert`** new events
    - Effectively "set clock ahead to next event"

Finding a good data structure

- Will show an efficient, non-obvious data structure
  - But first let's analyze some "obvious" ideas for *n* data items
  - All times worst-case; assume arrays "have room"

data	*insert algorithm / time*	*deleteMin algorithm / time*
unsorted array		
unsorted linked list		
sorted circular array		
sorted linked list		
binary search tree		
AVL tree		

Need a good data structure!

- Will show an efficient, non-obvious data structure for this ADT
  - But first let's analyze some "obvious" ideas for *n* data items
  - All times worst-case; assume arrays "have room"

data	*insert algorithm / time*		*deleteMin algorithm / time*	
unsorted array	add at end	$O(1)$	search	$O(n)$
unsorted linked list	add at front	$O(1)$	search	$O(n)$
sorted circular array	search / shift	$O(n)$	move front	$O(1)$
sorted linked list	put in right place	$O(n)$	remove at front	$O(1)$
binary search tree	put in right place	$O(n)$	leftmost	$O(n)$
AVL tree	put in right place	$O(\log n)$	leftmost	$O(\log n)$

More on possibilities

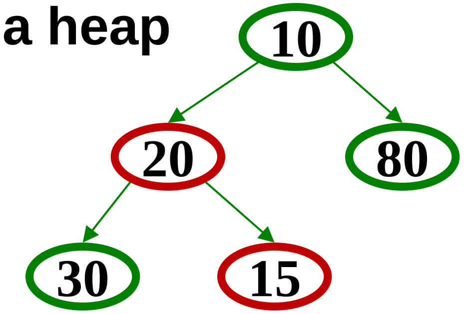
- *If* priorities are random, binary search tree will likely do better
  - O(`log` n) **`insert`** and O(`log` n) **`deleteMin`** on *average*
- One more idea: if priorities are 0, 1, …, k can use array of lists
  - **`insert`**: add to front of list at **`arr[priority]`**, $O(1)$
  - **`deleteMin`**: remove from lowest non-empty list $O(k)$
- We are about to see a data structure called a "binary heap"
  - O(`log` n) **`insert`** and O(`log` n) **`deleteMin`** *worst-case*
    - Possible because we don't support unneeded operations; no need to maintain a full sort
  - *Very* good constant factors
  - *If* items arrive in random order, then **`insert`** is $O(1)$ on *average*

Our data structure

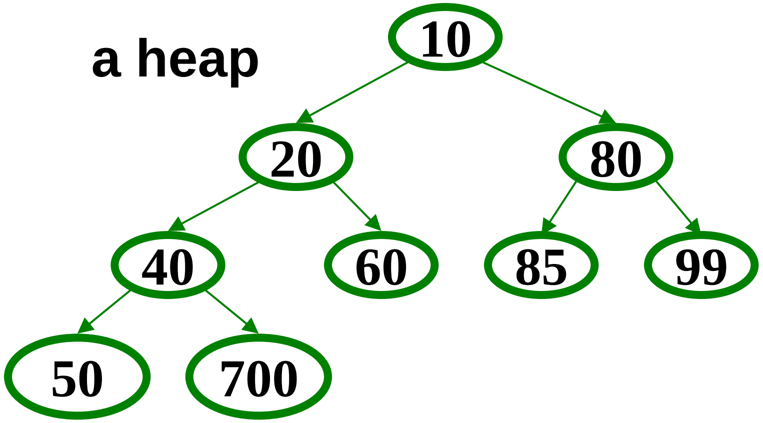
A *binary min-heap* (or just *binary heap* or just *heap*) is:

- Structure property: A *complet*e binary tree
- Heap property: The priority of every (non-root) node is greater than the priority of its parent
  - ***Not*** a binary search tree

not a heap



a heap

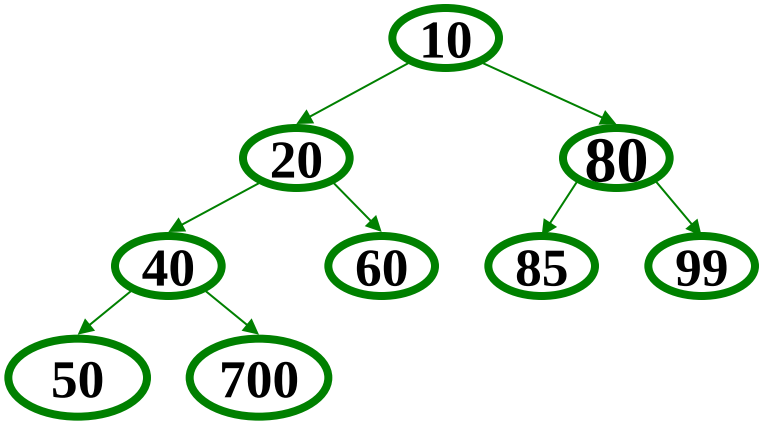


So:

- Where is the highest-priority item?
- What is the height of a heap with *n* items?

Operations: basic idea

- **`findMin`**: return **`root.data`**
- **`deleteMin`**:
  1. **`answer = root.data`**
  2. Move right-most node in last row to root to restore structure property
  3. “Percolate down” to restore heap property
- **`insert:`**
  1. Put new node in next position on bottom row to restore structure property
  2. “Percolate up” to restore heap property

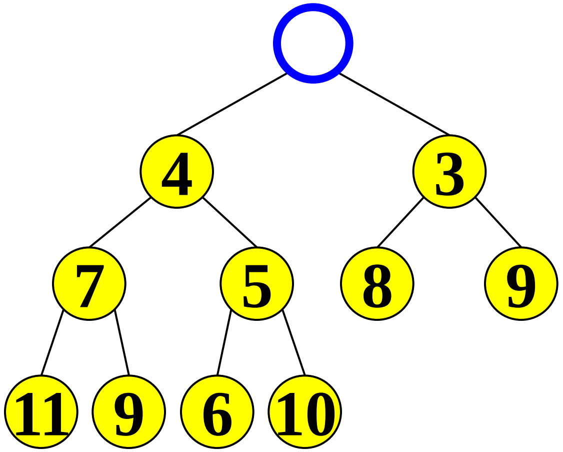


Overall strategy:

- *Preserve structure property*
- *Break and restore heap property*

DeleteMin

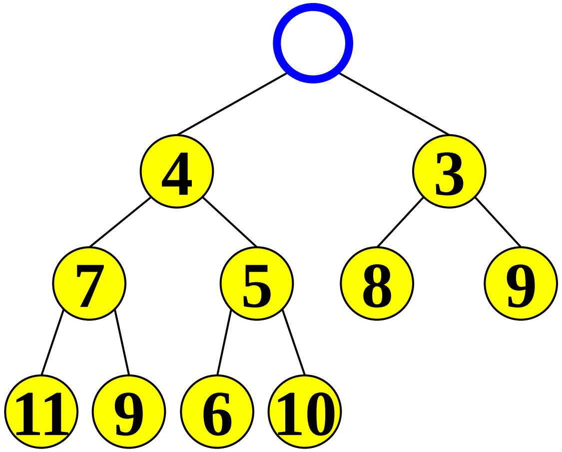
1. Delete (and later return) value at root node

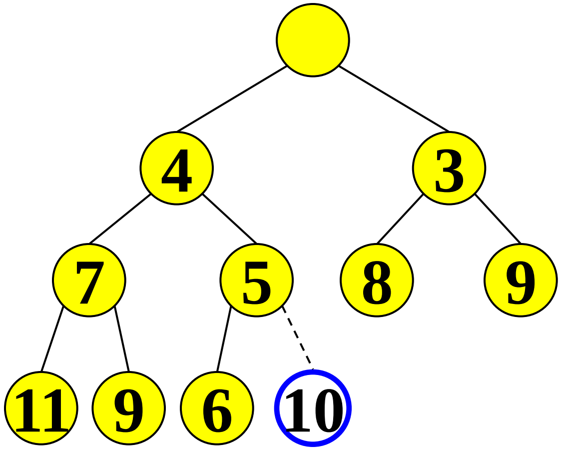


2. Restore the Structure Property

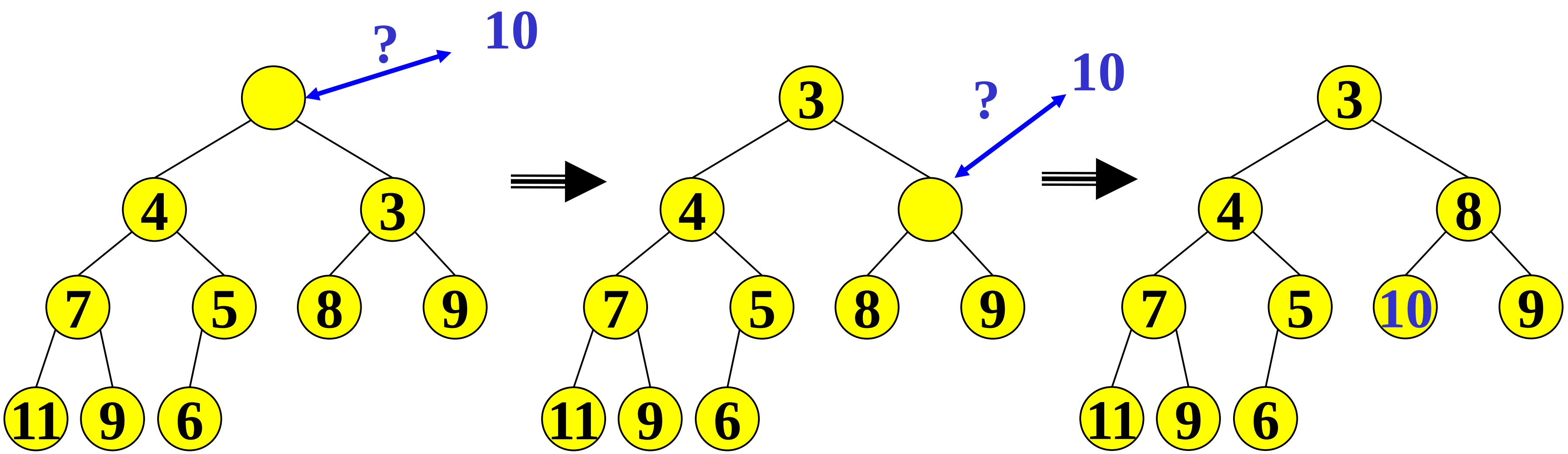
- We now have a “hole” at the root
  - Need to fill the hole with another value

- When we are done, the tree will have one less node and must still be complete





3. Restore the Heap Property



Percolate down:
- Keep comparing with both children
- Swap with lesser child and go down one level
- Done if both children are ≥ item or reached a leaf node

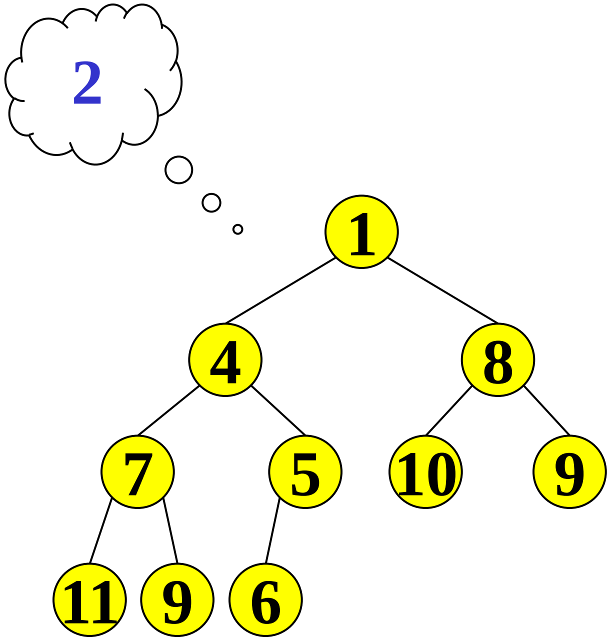
Why is this correct? What is the run time?

DeleteMin: Run Time Analysis

- Run time is *O*(height of heap)
- A heap is a complete binary tree
- Height of a complete binary tree of *n* nodes?
  - height = $\lfloor \mathtt{log}_2(n) \rfloor$
- Run time of **`deleteMin`** is $O(\mathtt{log}\ n)$

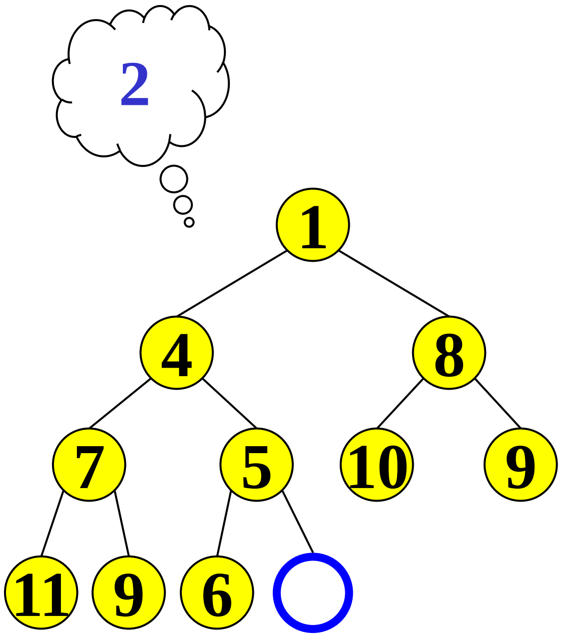
Insert

- Add a value to the tree
- Afterwards, structure and heap properties must still be correct

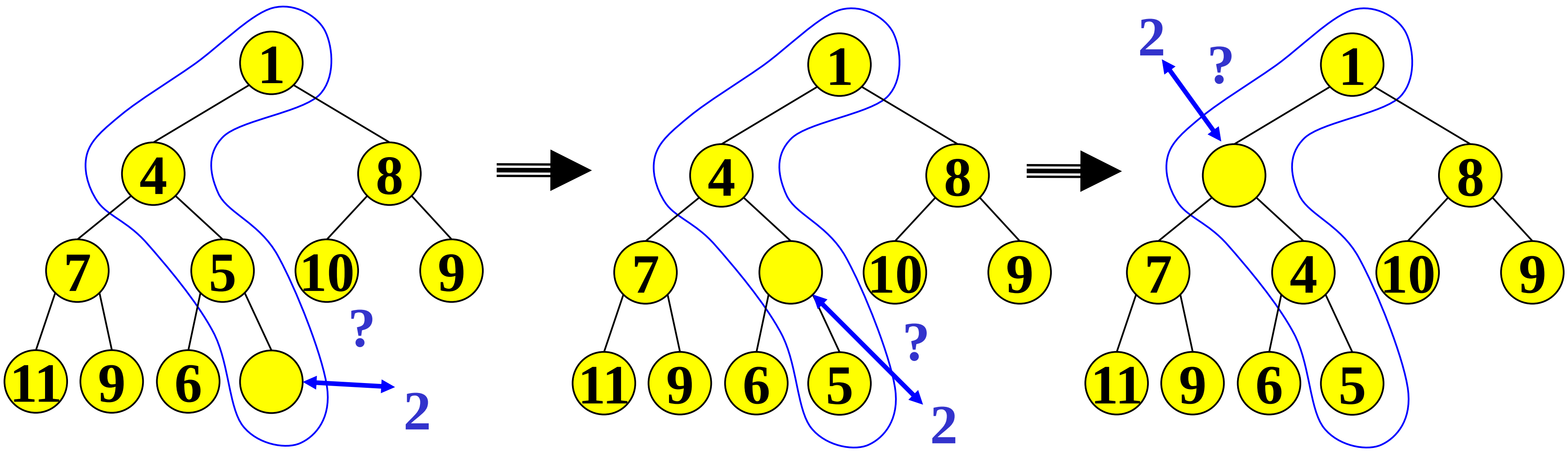


Insert: Maintain the Structure Property

- There is only one valid tree shape after we add one more node
- So put our new data there and then focus on restoring the heap property



Maintain the heap property



Percolate up:

- Put new data in new location
- If parent larger, swap with parent, and continue
- Done if parent ≤ item or reached root

Why is this correct? What is the run time?

Insert: Run Time Analysis

- Like **`deleteMin`**, worst-case time proportional to tree height
  - $O($**`log`** $n)$

- But… **`deleteMin`** needs the "last used" complete-tree position and **`insert`** needs the "next to use" complete-tree position
  - If "keep a reference to there" then **`insert`** and **`deleteMin`** have to adjust that reference: $O($**`log`** $n)$ in worst case
  - Could calculate how to find it in $O($**`log`** $n)$ from the root given the size of the heap
    - But it's not easy
    - And then **`insert`** is always $O($**`log`** $n)$, promised $O(1)$ on average (assuming random arrival of items)

- There's a "trick": don't represent complete trees with explicit edges!