CSE373: Data Structures & Algorithms

Lecture 4: Dictionaries; Binary Search Trees

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Fall 2013

Where we are

Studying the absolutely essential ADTs of computer science and classic data structures for implementing them

ADTs so far:

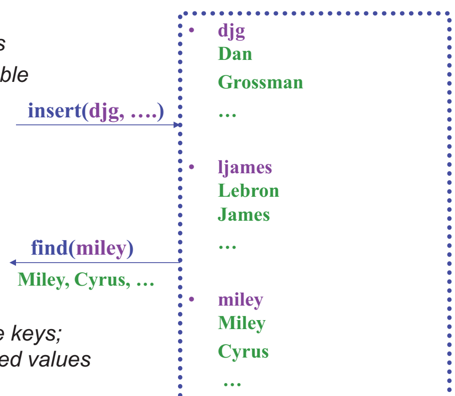
1. Stack: `push`, `pop`, `isEmpty`, …
2. Queue: `enqueue`, `dequeue`, `isEmpty`, …

Next:

3. Dictionary (a.k.a. Map): associate keys with values
   – Extremely common

The Dictionary (a.k.a. Map) ADT

- Data:
  – *set* of (key, value) *pairs*
  – keys must be *comparable*
- Operations:
  – `insert(key,value)`
  – `find(key)`
  – `delete(key)`
  – …



Will tend to emphasize the keys; don't forget about the stored values

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Comparison: The Set ADT

The *Set* ADT is like a Dictionary without any values
– A key is *present* or not (no repeats)

For `find`, `insert`, `delete`, there is little difference
– In dictionary, values are "just along for the ride"
– So *same data-structure ideas* work for dictionaries and sets

But if your Set ADT has other important operations this may not hold
– `union`, `intersection`, `is_subset`
– Notice these are binary operators on sets

Dictionary data structures

There are many good data structures for (large) dictionaries

1. AVL trees
   – Binary search trees with *guaranteed balancing*
2. B-Trees
   – Also always balanced, but different and shallower
   – B!=Binary; B-Trees generally have large branching factor
3. Hashtables
   – Not tree-like at all

Skipping: Other balanced trees (e.g., red-black, splay)

But first some applications and less efficient implementations…

A Modest Few Uses

Any time you want to store information according to some key and be able to retrieve it efficiently
– Lots of programs do that!

- Search: inverted indexes, phone directories, …
- Networks: router tables
- Operating systems: page tables
- Compilers: symbol tables
- Databases: dictionaries with other nice properties
- Biology: genome maps
- …

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list			
• Unsorted array			
• Sorted linked list			
• Sorted array			

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$*	$O(n)$	$O(n)$
• Unsorted array	$O(1)$*	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\log n)$	$O(n)$

* Unless we need to check for duplicates

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

Lazy Deletion

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making **`delete`** as fast as **`find`**:
- Instead of actually removing the item just mark it deleted

Plusses:
- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

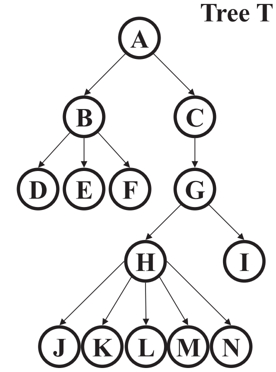
Minuses:
- Extra *space* for the "is-it-deleted" flag
- Data structure full of deleted nodes wastes *space*
- **`find`** $O(\log m)$ *time* where *m* is data-structure size (okay)
- May complicate other operations

Tree terms (review?)

***root*(tree)**
***leaves*(tree)**
***children*(node)**
***parent*(node)**
***siblings*(node)**
***ancestors*(node)**
***descendents*(node)**
***subtree*(node)**

***depth*(node)**
***height*(tree)**
***degree*(node)**
***branching factor*(tree)**

Tree T



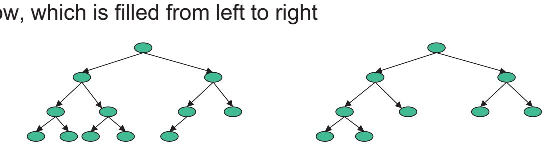
Some tree terms (mostly review)

- There are many kinds of trees
  - Every binary tree is a tree
  - Every list is kind of a tree (think of "next" as the one child)
- There are many kinds of binary trees
  - Every binary search tree is a binary tree
  - Later: A binary heap is a different kind of binary tree
- A tree can be balanced or not
  - A balanced tree with *n* nodes has a height of $O(\log n)$
  - Different tree data structures have different "balance conditions" to achieve this

Kinds of trees

Certain terms define trees with specific structure

- Binary tree: Each node has at most 2 children (branching factor 2)
- *n*-ary tree: Each node has at most *n* children (branching factor *n*)
- Perfect tree: Each row completely full
- Complete tree: Each row completely full except maybe the bottom row, which is filled from left to right

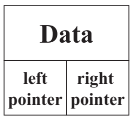


What is the height of a perfect binary tree with n nodes?
A complete binary tree?

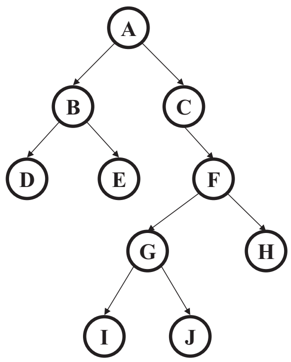
Binary Trees

- Binary tree is empty or
  - A root *(with data)*
  - A left subtree *(may be empty)*
  - A right subtree *(may be empty)*

- Representation:



- For a dictionary, data will include a key and a value



Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:
- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:
- max # of leaves: 2^h
- max # of nodes: $2^{(h+1)} - 1$
- min # of leaves: 1
- min # of nodes: $h + 1$

For n nodes, we cannot do better than O(`log` n) height, and we want to avoid O(n) height

Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {

            ???

}
```


Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {
  if(root == null)
    return -1;
  return 1 + max(treeHeight(root.left),
                 treeHeight(root.right));
}
```

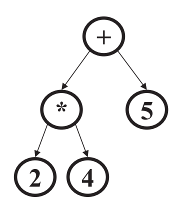
Running time for tree with *n* nodes: *O*(*n*) – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree
- *In-order*: left subtree, root, right subtree
- *Post-order*: left subtree, right subtree, root

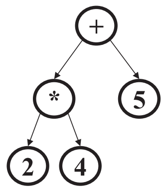


(an expression tree)

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

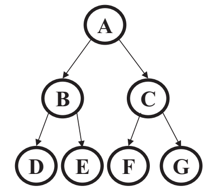
- *Pre-order*: root, left subtree, right subtree
  + * 2 4 5
- *In-order*: left subtree, root, right subtree
  2 * 4 + 5
- *Post-order*: left subtree, right subtree, root
  2 4 * 5 +



(an expression tree)

More on traversals

```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
    process(t.element);
    inOrderTraversal(t.right);
  }
}
```



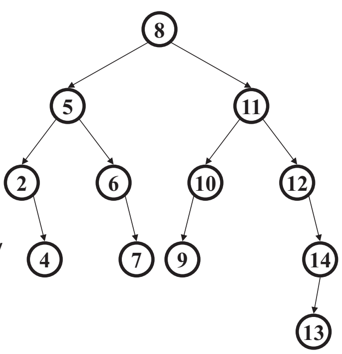
Sometimes order doesn't matter
- Example: sum all elements

Sometimes order matters
- Example: print tree with parent above indented children (pre-order)
- Example: evaluate an expression tree (post-order)

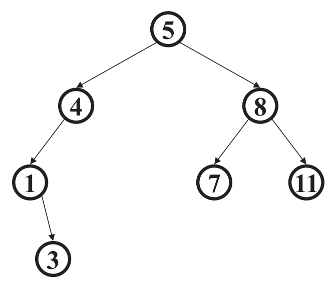
```
A
 B
  D
  E
 C
  F
  G
```

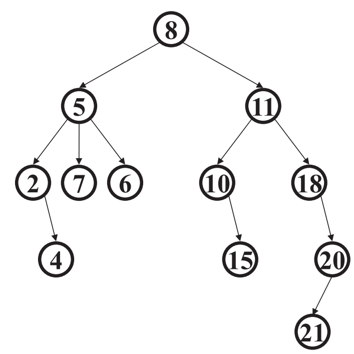

Binary Search Tree

- Structure property ("binary")
  - Each node has ≤ 2 children
  - Result: keeps operations simple
- Order property
  - All keys in left subtree smaller than node's key
  - All keys in right subtree larger than node's key
  - Result: easy to find any given key

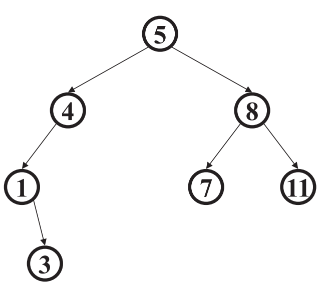


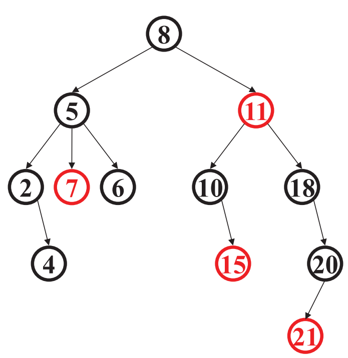
Are these BSTs?



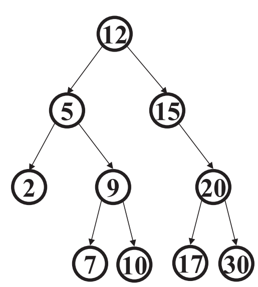


Are these BSTs?



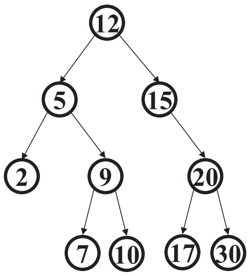


Find in BST, Recursive




```
Data find(Key key, Node root){
 if(root == null)
   return null;
 if(key < root.key)
   return find(key,root.left);
 if(key > root.key)
   return find(key,root.right);
 return root.data;
}
```

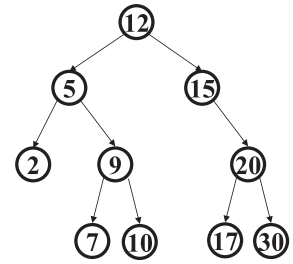
Find in BST, Iterative



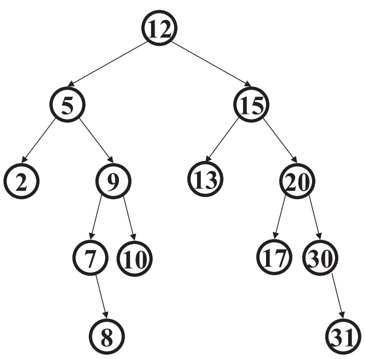

```
Data find(Key key, Node root){
 while(root != null
       && root.key != key) {
  if(key < root.key)
    root = root.left;
  else(key > root.key)
    root = root.right;
 }
 if(root == null)
    return null;
 return root.data;
}
```


Other "Finding" Operations

- Find *minimum* node
  - "the liberal algorithm"
- Find *maximum* node
  - "the conservative algorithm"
- Find *predecessor* of a non-leaf
- Find *successor* of a non-leaf
- Find *predecessor* of a leaf
- Find *successor* of a leaf



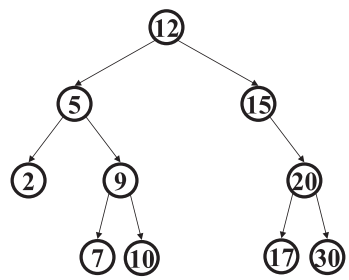
Insert in BST




```
insert(13)
insert(8)
insert(31)
```

(New) insertions happen only at leaves – easy!

Deletion in BST



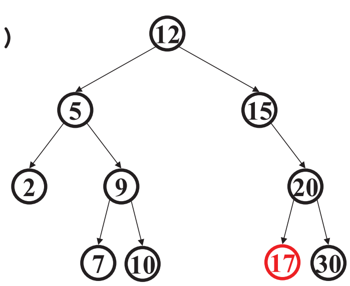
Why might deletion be harder than insertion?

Deletion

- Removing an item disrupts the tree structure
- Basic idea: `find` the node to be removed, then "fix" the tree so that it is still a binary search tree
- Three cases:
  - Node has no children (leaf)
  - Node has one child
  - Node has two children

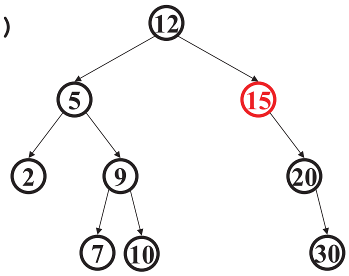
Deletion – The Leaf Case

```
delete(17)
```



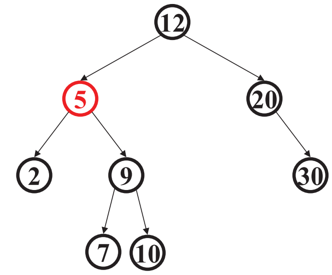
Deletion – The One Child Case

`delete(15)`



Deletion – The Two Child Case

`delete(5)`



What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

Options:
- *successor* from right subtree: **`findMin(node.right)`**
- *predecessor* from left subtree: **`findMax(node.left)`**
  - These are the easy cases of predecessor/successor

Now delete the original node containing *successor* or *predecessor*
- Leaf or one child case – easy cases of delete!

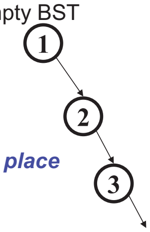
Lazy Deletion

- Lazy deletion can work well for a BST
  - Simpler
  - Can do “real deletions” later as a batch
  - Some inserts can just “undelete” a tree node
- But
  - Can waste space and slow down find operations
  - Make some operations more complicated:
    - How would you change **`findMin`** and **`findMax`**?

BuildTree for BST

- Let’s consider **`buildTree`**
  - Insert all, starting from an empty tree
- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
  - If inserted in given order, what is the tree?
  - What big-O runtime for this kind of sorted input?
  - Is inserting in the reverse order any better?

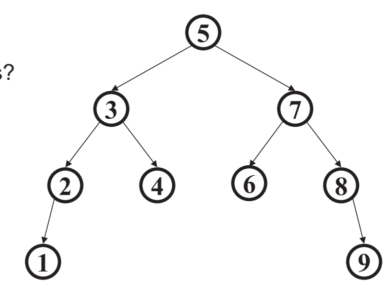
$O(n^2)$
Not a happy place



BuildTree for BST

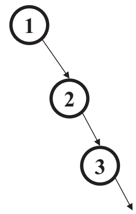
- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
- What we if could somehow re-arrange them
  - median first, then left median, right median, etc.
  - 5, 3, 7, 2, 1, 4, 8, 6, 9
  - What tree does that give us?
  - What big-O runtime?

O(n log n), definitely better



Unbalanced BST

- Balancing a tree at build time is insufficient, as sequences of operations can eventually transform that carefully balanced tree into the dreaded list
- At that point, everything is *O(n)* and nobody is happy
  - `find`
  - `insert`
  - `delete`



Balanced BST

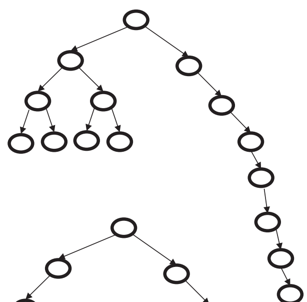
Observation
- BST: the shallower the better!
- For a BST with *n* nodes inserted in arbitrary order
  - Average height is $O(\log n)$ – see text for proof
  - Worst case height is $O(n)$
- Simple cases, such as inserting in key order, lead to the worst-case scenario

Solution: Require a **Balance Condition** that
1. Ensures depth is always $O(\log n)$ – strong enough!
2. Is efficient to maintain – not too strong!

Potential Balance Conditions

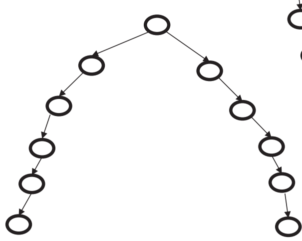
1. Left and right subtrees of the *root* have equal number of nodes

   Too weak!
   Height mismatch example:



2. Left and right subtrees of the *root* have equal *height*

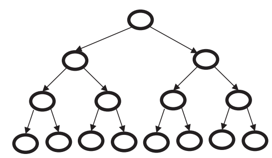
   Too weak!
   Double chain example:



Potential Balance Conditions

3. Left and right subtrees of every node have equal number of nodes

   Too strong!
   Only perfect trees (2^n – 1 nodes)



4. Left and right subtrees of every node have equal *height*

   Too strong!
   Only perfect trees (2^n – 1 nodes)

The AVL Balance Condition

Left and right subtrees of *every node* have *heights* **differing by at most 1**

Definition: **balance**(*node*) = height(*node*.left) – height(*node*.right)

AVL *property*: **for every node x, $-1 \le$ balance(x) ≤ 1**

- Ensures small depth
  - Will prove this by showing that an AVL tree of height *h* must have a number of nodes *exponential* in *h*
- Efficient to maintain
  - Using single and double rotations