CSE 373: Data Structures and Algorithms

Lecture 17: Graphs
What are graphs?

• Yes, this is a graph....

• But we are interested in a different kind of “graph”
Airline Routes

Nodes = cities
Edges = direct flights
Computer Networks

Nodes = computers
Edges = transmission rates
Nodes = courses
Directed edge = prerequisite
Graphs

- **graph**: a data structure containing
  - a set of vertices $V$
  - a set of edges $E$, where an edge represents a connection between 2 vertices
  - $G = (V, E)$
  - edge is a pair $(v, w)$ where $v, w$ in $V$

- the graph at right: $V = \{a, b, c\}$ and $E = \{(a, b), (b, c), (c, a)\}$

- Assuming that a graph can only have one edge between a pair of vertices and cannot have an edge to itself, what is the maximum number of edges a graph can contain, relative to the size of the vertex set $V$?
• **path**: a path from vertex $A$ to $B$ is a sequence of edges that can be followed starting from $A$ to reach $B$
  – can be represented as vertices visited or edges taken
  – example: path from $V$ to $Z$: \{b, h\} or \{V, X, Z\}

• **reachability**: $v_1$ is *reachable* from $v_2$ if a path exists from $V_1$ to $V_2$

• **connected** graph: one in which it's possible to reach any node from any other
  – is this graph connected?
Cycles

• **cycle**: path from one node back to itself
  – example: \{b, g, f, c, a\} or \{V, X, Y, W, U, V\}

• **loop**: edge directly from node to itself
  – many graphs don't allow loops
Weighted graphs

- **weight**: (optional) cost associated with a given edge

- example: graph of airline flights
  - if we were programming this graph, what information would we have to store for each vertex / edge?
Directed graphs

- directed graph (digraph): edges are one-way connections between vertices
  - if graph is directed, a vertex has a separate $\text{in/out degree}$
Trees as Graphs

• Every tree is a graph with some restrictions:
  – the tree is directed
  – there is exactly one directed path from the root to every node
More terminology

• **degree**: number of edges touching a vertex
  – example: W has degree 4
  – what is the degree of X? of Z?

• **adjacent vertices**: connected directly by an edge
Graph questions

• Are the following graphs directed or not directed?
  – Buddy graphs of instant messaging programs? (vertices = users, edges = user being on another's buddy list)
  – bus line graph depicting all of Seattle's bus stations and routes
  – graph of movies in which actors have appeared together

• Are these graphs potentially cyclic? Why or why not?
Graph exercise

• Consider a graph of instant messenger buddies.
  – What do the vertices represent? What does an edge represent?
  – Is this graph directed or undirected? Weighted or unweighted?
  – What does a vertex's degree mean? In degree? Out degree?
  – Can the graph contain loops? cycles?

• Consider this graph data:
  – Jessica's buddy list: Meghan, Alan, Martin.
  – Meghan's buddy list: Alan, Lori.
  – Toni's buddy list: Lori, Meghan.
  – Martin's buddy list: Lori, Meghan.
  – Alan's buddy list: Martin, Jessica.
  – Lori's buddy list: Meghan.

  – Compute the in/out degree of each vertex. Is the graph connected?
  – Who is the most popular? Least? Who is the most antisocial?
  – If we're having a party and want to distribute the message the most quickly, who should we tell first?