Priority Queues: Binary Min Heaps

CSE 373
Data Structures and Algorithms

Today’s Outline

• Announcements
  – Midterm #1, next Friday Oct 21.
  – Assignment #3 coming soon, not due until Oct 27.

• Today’s Topics:
  – Dictionary
    • Balanced Binary Search Trees - (AVL Trees)
  – Priority Queues
    • Binary Min Heap

Priority Queue ADT

1. **PQueue data**: collection of data with priority

2. **PQueue operations**
   – insert
   – deleteMin
   (also: create, destroy, is_empty)

3. **PQueue property**: for two elements in the queue, x and y, if x has a lower priority value than y, x will be deleted before y

Applications of the Priority Q

• Select print jobs in order of decreasing length
• Forward packets on network routers in order of urgency
• Select most frequent symbols for compression
• Sort numbers, picking minimum first

• Anything greedy

Implementations of Priority Queue ADT

<table>
<thead>
<tr>
<th></th>
<th>insert</th>
<th>deleteMin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsorted list (Array)</td>
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<tr>
<td>Unsorted list (Linked-List)</td>
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<tr>
<td>Sorted list (Array)</td>
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<tr>
<td>Sorted list (Linked-List)</td>
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<tr>
<td>Binary Search Tree (BST)</td>
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Representing Complete Binary Trees in an Array

From node i:
left child:  
right child: parent:

implicit (array) implementation:

```
A B C D E F G H I J K L
0 1 2 3 4 5 6 7 8 9 10 11 12 13
```
Why better than tree with pointers?

Heap Order Property

Heap order property: For every non-root node X, the value in the parent of X is less than (or equal to) the value in X.

Heap Operations

- findMin:
- insert(val): percolate up.
- deleteMin: percolate down.

Heap – Insert(val)

Basic Idea:
1. Put val at “next” leaf position
2. Repeatedly exchange node with its parent if needed

Insert pseudo Code (optimized)

```java
void insert(Object o) {
    assert(!isFull());
    size++;
    newPos = percolateUp(size, o);
    Heap[newPos] = o;
}
```

Insert: percolate up

```
int percolateUp(int hole, Object val) {
    while (hole > 1 && val < Heap[hole/2]) {
        Heap[hole] = Heap[hole/2];
        hole /= 2;
    }
    return hole;
}
```

runtime:

(Java code in book)
Heap – DeleteMin

Basic Idea:
1. Remove root (that is always the min!)
2. Put “last” leaf node at root
3. Find smallest child of node
4. Swap node with its smallest child if needed.
5. Repeat steps 3 & 4 until no swaps needed.

DeleteMin pseudo Code (Optimized)

```java
Object deleteMin() {
    assert(!isEmpty());
    returnVal = Heap[1];
    size--;
    newPos = percolateDown(1, Heap[size+1]);
    Heap[newPos] = Heap[size + 1];
    return returnVal;
}
```

```java
int percolateDown(int hole, Object val) {
    assert(2*hole <= size);
    left = 2*hole;
    right = left + 1;
    if (right <= size && Heap[right] < Heap[left])
        target = right;
    else
        target = left;
    if (Heap[target] < val) {
        Heap[hole] = Heap[target];
        hole = target;
        percolateDown(hole, val);
    }
    return hole;
}
```

DeleteMin: percolate down

[Image of a heap with numbers and nodes marked for percolation]

Other Priority Queue Operations

- **decreaseKey**
  - given a pointer to an object in the queue, reduce its priority value
  
  Solution: change priority and ______________________

- **increaseKey**
  - given a pointer to an object in the queue, increase its priority value
  
  Solution: change priority and ______________________

Why do we need a pointer? Why not simply data value?

Other Heap Operations

- **decreaseKey(objPtr, amount)**: raise the priority of a object, percolate up
- **increaseKey(objPtr, amount)**: lower the priority of a object, percolate down
- **remove(objPtr)**: remove an object, move to top, then delete.
  1) decreaseKey(objPtr, ∞)
  2) deleteMin()

Worst case Running time for all of these:
FindMax?
ExpandHeap – when heap fills, copy into new space.
Binary Min Heaps (summary)

- **insert**: percolate up. $\Theta(\log N)$ time.
- **deleteMin**: percolate down. $\Theta(\log N)$ time.
- **Build Heap**?

BuildHeap: Floyd’s Method

Add elements arbitrarily to form a complete tree.
Pretend it’s a heap and fix the heap-order property!

Buildheap pseudocode

```java
private void buildHeap() {
    for (int i = currentSize/2; i > 0; i--)
        percolateDown(i);
}
```

Finally…