CSE 373, Autumn 2008, Assignment 2 Solutions

October 18, 2008

1. (8 points)
   (a) \( n = ((795 - (-10))/7) + 1 = 116 \)
       Sum = \((((-10) + 795)116)/2 = 45530 \)
   (b) Sum = \((256 * (1 - (1/2)^9))/(1 - (1/2)) = 511 \)
   (c) Sum = \((1 * (3^9 - 1))/(3 - 1) = 9841 \)
   (d) Sum = \(144/(1 - (1/4)) = 192 \)

2. (6 points)
   (a) \(10^{x+y+z} \)
   (b) \(xy \)
   (c) \(1 + 2 \log_2 x + 3 \log_2 y \)

3. (7 points)
   Basis Step:
   \( n = 1, (1 + 1) = 1 * (1 + 3)/2 = 2. \)
   Induction hypothesis:
   \( \sum_{i=1}^{k} (i + 1) = \frac{k(k+3)}{2}, \text{ for some } k. \)
   Induction step:
   \( \sum_{i=1}^{k+1} (i + 1) = \sum_{i=1}^{k} (i + 1) + ((k + 1) + 1) = \frac{k(k+3)}{2} + ((k + 1) + 1) = \)
   This represents the proposition to be proved for the case \( n = k + 1, \) and completes the proof.

4. (6 points)
   (a) \{\}, \{0\}, \{1\}, \{0,1\}
   (b) \{(0,0), (0,1), (1,0), (1,1)\}
   (c) \{(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)\}

1
5. (18 points)

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
<th>R6</th>
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</thead>
<tbody>
<tr>
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<tr>
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</table>

6. (20 points, 15 for table entries and 5 for explanations)

<table>
<thead>
<tr>
<th></th>
<th>100</th>
<th>2n + 5</th>
<th>(\log_2 n)</th>
<th>(5n^2)</th>
<th>(n \log_2 n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3n + 1)</td>
<td>(\Omega)</td>
<td>(\Theta)</td>
<td>(\Omega)</td>
<td>(O)</td>
<td>(O)</td>
</tr>
<tr>
<td>(0.001 * 2^{n-10})</td>
<td>(\Omega)</td>
<td>(\Omega)</td>
<td>(\Omega)</td>
<td>(\Omega)</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(\log_{10} n^2)</td>
<td>(\Omega)</td>
<td>(\Omega)</td>
<td>(\Omega)</td>
<td>(O)</td>
<td>(\Theta)</td>
</tr>
</tbody>
</table>

\(0.001 * 2^{n-10} \geq 5n^2\) for \(n \geq 33\) as can be verified by taking base 2 logs on both sides.

\[\log_{10} n^n = n \log_{10} n = n \log_2 n / \log_2 10 = \Theta(n \log_2 n)\]

7. (20 points)

(a) (12 points) We will use stack \(S_a\) for enqueueing, \(S_b\) for dequeueing, and a boolean variable \(enQmode\) for storing the current operating mode. The methods are shown below.

```java
boolean isEmpty(){
    if(Sa.isEmpty() && Sb.isEmpty())
        return true;
    else
        return false; }

void enqueue(Object obj){
    if(!enQmode){
        while(!Sb.isEmpty())
            Sa.push(Sb.pop());}
    Sa.push(obj); }

Object dequeue(){
    if(enQmode){
        while(!Sa.isEmpty())
            Sb.push(Sa.pop());}
    return Sb.pop(); }
```
(b) (4 points) The isEmpty method is constant time. The enqueue and dequeue operations take $O(m)$ time in the worst case where $m$ is the current size of the queue. This is because we may need to move all $m$ objects from one stack to another.

(c) (4 points) The total time complexity is $O(n^2)$. There are $2n$ operations each of which takes $O(n)$ time.

8. (15 points)

(a) (5 points) The algorithm goes thru the following steps.

```
poly = 4
poly = 4 * 3 + 8 = 20
poly = 20 * 3 + 0 = 60
poly = 60 * 3 + 1 = 181
poly = 181 * 3 + 2 = 545
```

(b) (5 points) Observe that the polynomial $a_0 + a_1x + a_2x^2 + \ldots$ can be rewritten as $a_0 + x(a_1 + x(a_2 + \ldots)$ by repeatedly factoring out $x$. The algorithm computes the polynomial using this equivalent form.

(c) (5 points) The running time is $\Theta(n)$. 