Notation

\[ x \downarrow \text{ floor function: the largest integer } \leq x \]
\[ x \uparrow \text{ ceiling function: the smallest integer } \geq x \]

Positional Number System

In base 10, an unsigned integer is \( x = \sum_{i=0}^{n} a_i 10^i \), where \( a_i \) is a digit from 0 to 9.
In base 2 (binary) an unsigned integer is \( x = \sum_{i=0}^{n} a_i 2^i \), where \( a_i \) is a bit value, 0 or 1.

Logs

\[ \log_2 x = y \text{ means } x = 2^y \]
\[ \log(x,y) = \log x \log y; \quad \log(x/y) = \log x - \log y; \quad \log(x^y) = y \log x \]
\[ \log \log x < \log x < x \text{ for all } x > 0 \]
\[ \log_2 a = \frac{\log_2 x}{\log_2 y} \]

Series

\[ S(n) = 1 + 2 + 3 + \ldots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]
\[ C(n) = 1 + 2^2 + 3^2 + \ldots + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]
\[ H(n), \text{ the } n^{th} \text{ harmonic number } = 1 + 1/2 + 1/3 + \ldots + 1/n = \ln n + c \text{ where } \ln n \text{ is the natural logarithm of } n \text{ and } c \text{ is a constant. Thus } H(n) = O(\log n) \]

The Big-Oh Notation

\( T(n) = O(f(n)) \) if there are constants \( c \) and \( n_0 \) such that \( T(n) \leq c \cdot f(n) \) for all \( n \geq n_0 \)

Constant time \( O(1) \)
Logarithmic time \( O(\log n) \)
Linear time \( O(n) \)
\( O(n \log n) \) grows faster than linear time but not as fast as quadratic time
Quadratic time \( O(n^2) \)
Cubic time is \( O(n^3) \)
Polynomial time is \( O(n^k) \) for some \( k \)
Exponential time is \( O(c^n) \) for some \( c > 1 \)