## CSE 373 Lecture 6: Trees

- ✦ Today's agenda:
  - ➪ Trees: Definition and terminology
  - ➪ Traversing trees
  - ➪ Binary search trees
  - ➪ Inserting into and deleting from trees
- ✦ Covered in Chapter 4 of the text



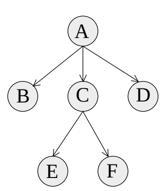
## Why Do We Need Trees?

- ✦ Lists, Stacks, and Queues represent linear sequences
- ✦ Data often contain hierarchical relationships that cannot be expressed as a linear ordering
  - ➪ File directories or folders on your computer
  - ➪ Moves in a game
  - ➪ Employee hierarchies in organizations and companies
  - ➪ Family trees
  - ➪ Classification hierarchies (e.g. phylum, family, genus, species)



## Tree Jargon

- ✦ Basic terminology:
  - nodes and edges
  - root
  - subtrees
  - parent
  - children, siblings
  - leaves
  - path
  - ancestors
  - descendants
  - path length

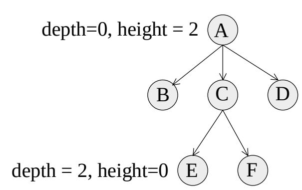



Note: Arrows denote directed edges
Trees always contain directed edges
but arrows are often omitted.



## More Tree Jargon

- ✦ Length of a path = number of edges
- ✦ Depth of a node N = length of path from root to N
- ✦ Height of node N = length of longest path from N to a leaf
- ✦ Depth and height of tree = ?



## Definition and Tree Trivia

- Recursive Definition of a Tree:

A tree is a set of nodes that is
  a. an empty set of nodes, or
  b. has one node called the root from which zero or more trees (subtrees) descend.

- A tree with N nodes always has ___ edges
- Two nodes in a tree have at most how many paths between them?
- Can a non-zero path from node N reach node N again?
- Does depth of nodes in a non-zero path increase or decrease?



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- Recursive Definition of a Tree:

A tree is a set of nodes that is
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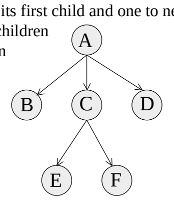
- A tree with N nodes always has N-1 edges
- Two nodes in a tree have at most one path between them
- Can a non-zero path from node N reach node N again?
  - No! Trees can never have cycles.
- Does depth of nodes in a non-zero path increase or decrease?
  - Depth always increases in a non-zero path



## Implementation of Trees

- Obvious Pointer-Based Implementation: Node with value and pointers to children
  - Problem: Do not usually know number of children for a node in advance. How many pointers should we allocate space for?
- Better Implementation: 1st Child/Next Sibling Representation
  - Each node has 2 pointers: one to its first child and one to next sibling
  - Can handle arbitrary number of children
  - Exercise: Draw the representation for this tree…






## Application: Arithmetic Expression Trees

Example Arithmetic Expression:

A + (B * (C / D) )

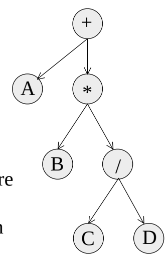
How would you express this as a tree?

## Application: Arithmetic Expression Trees

Example Arithmetic Expression:

A + (B * (C / D) )

Tree for the above expression:

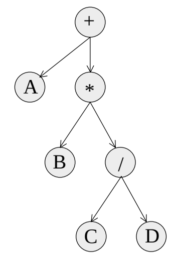



- Used in most compilers
- No parenthesis need – use tree structure
- Can speed up calculations e.g. replace / node with C/D if C and D are known
- Calculate by traversing tree (how?)



## Traversing Trees

- Preorder: Root, then Children
  - + A * B / C D
- Postorder: Children, then Root
  - A B C D / * +
- Inorder: Left child, Root, Right child
  - A + B * C / D






## Example Code for Recursive Preorder

```
void print_preorder ( TreeNode T)
{ TreeNode P;
  if  ( T == NULL ) return;
  else  {  print_element(T-> Element);
           P = T -> FirstChild;
           while (P != NULL) {
            print_preorder ( P );
            P = P -> NextSibling; }
         }
}
```

What is the running time for a tree with N nodes?



## Preorder Traversal with a Stack

```
void Stack_Preorder (TreeNode T, Stack S)
{
if (T == NULL) return; else push(T,S);
while (!isempty(S)) {
   T = pop(S);
   print_element(T -> Element);
   if  (T -> Right !=  NULL) push(T -> Right, S);
   if  (T -> Left !=  NULL)  push(T -> Left, S);
   }
}
```

What is the running time for a tree with N nodes?

## Binary Trees

- Every node has at most two children
  - Most popular tree in computer science
- Given N nodes, what is the minimum depth of a binary tree?
- What is the maximum depth of a binary tree?



## Binary Trees

- Every node has at most two children
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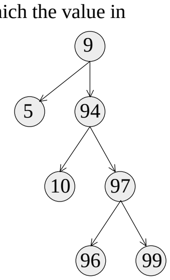
- Given N nodes, what is the minimum depth of a binary tree?
  - At depth d, you can have $N = 2^d$ to $2^{d+1}-1$ nodes (a full tree)
  - So, minimum depth d is: $\log N \leq d \leq \log(N+1)-1$ or $\Theta(\log N)$
- What is the maximum depth of a binary tree?
  - Degenerate case: Tree is a linked list!
  - Maximum depth = N-1
- Goal: Would like to keep depth at around log N to get better performance than linked list for operations like Find.



## Binary Search Trees

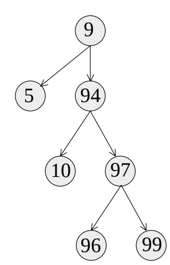
- Binary search trees are binary trees in which the value in every node is:
  - > all values in the node's left subtree
  - < all values in the node's right subtree
- Application: "Look-up" table
  - Example: Given SSN, return student record
  - SSN stored in each node as the key value
- Operations:
  - Find, FindMin, FindMax
  - Insert, Delete





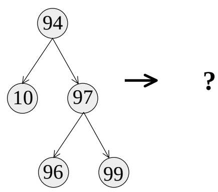

## Operations on Binary Search Trees

- How would you implement these?
  - Recursive definition of binary search trees allows recursive routines!
- Position FindMin(Tree T)
- Position FindMax(Tree T)
- Position Find(ElementType X, Tree T)
- Tree Insert(ElementType X, Tree T)
- Tree Delete(ElementType X, Tree T)



## Insert Operation

- Tree Insert(ElementType X, Tree T)
  - Do a “Find” operation for X
  - If X is found → update duplicates counter
  - Else, “Find” stops at a NULL pointer
  - Insert Node with X there!
- Example: Insert 95



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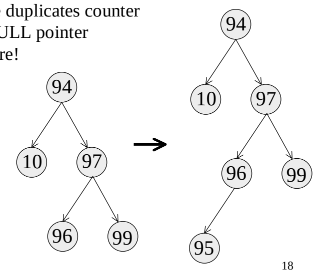
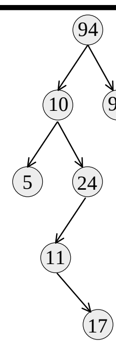





## Delete Operation

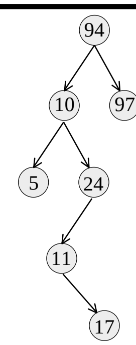
- Delete is a bit trickier…Why?
- Suppose you want to delete 10
- Strategy:
  - Find 10
  - Delete the node containing 10
- Problem: When you delete a node, what do you replace it by?



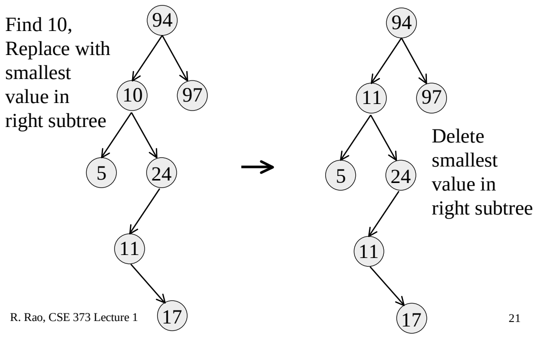



## Delete Operation

- Problem: When you delete a node, what do you replace it by?
- Solution:
  1. If it has no children, by NULL
  2. If it has 1 child, by that child
  3. If it has 2 children, by the node with the smallest value in its right subtree
- Examples:
  1. Delete 5
  2. Delete 24 (note: recursive deletion)
  3. Delete 10 (note: recursive deletion)



## Example: Delete “10”



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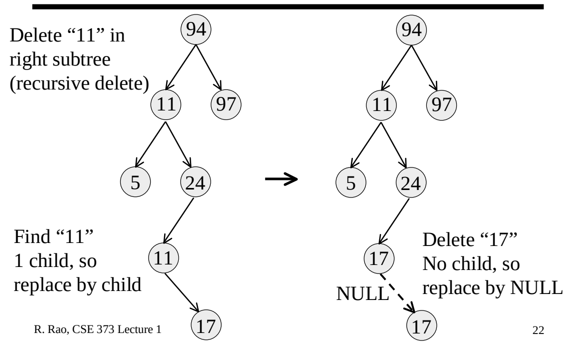




Next Class:

Analysis of Binary Search Tree Operations

Other Species of Trees: AVL, splay, and B-trees

Homework #2 will be assigned

To Do:

Read Chapter 4