CSE 373 Lecture 17: More Sorting

- Today's agenda:
  - Midterm solutions (on board)
  - The Fastest Sorting Algorithms:
    - Mergesort
    - Quicksort
- Covered in Chapter 7 of the textbook

Preorder Traversal with a Stack

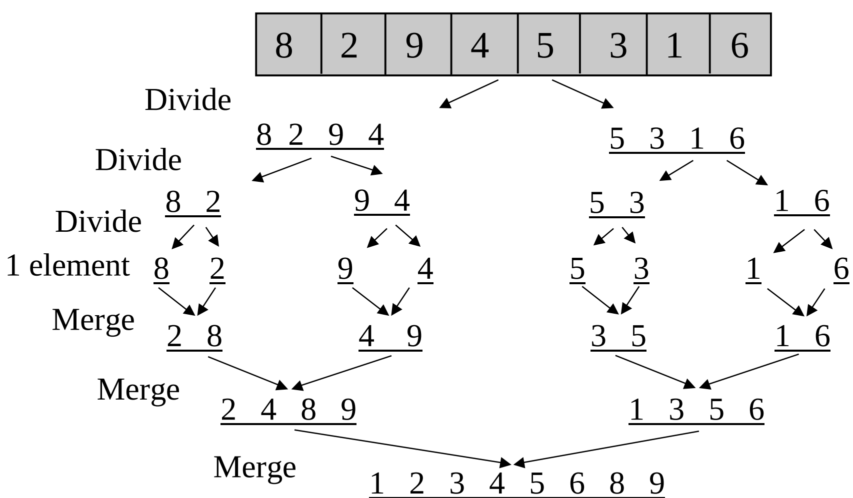
```
void Stack_Preorder (Tree T, Stack S)
{
if (T == NULL) return; else push(T,S);
while (!isempty(S)) {
   T = pop(S);
   print_element(T -> Element);
   if  (T -> Right !=  NULL) push(T -> Right, S);
   if  (T -> Left !=  NULL)  push(T -> Left, S);
   }
}
```

Recall from Last Time: Mergesort

- Based on the idea of “Divide and Conquer”:
  1. Divide problem into smaller parts
  2. Independently solve the parts
  3. Combine these solutions to get overall solution
- ***Mergesort***: Divide array into two halves, *recursively* sort left and right halves, then *merge* two halves
- Example: Mergesort the input array:

0	1	2	3	4	5	6	7
8	2	9	4	5	3	1	6

Mergesort Example



Mergesort Analysis

- ✦ Let T(N) be the running time for an array of N elements
- ✦ Mergesort divides array in half and calls itself on the two halves. After returning, it merges both halves using a temporary array (see textbook for code).
- ✦ Each recursive call takes T(N/2) and merging takes O(N)
- ✦ Therefore, the recurrence relation for T(N) is:
  - ➩ $T(1) = O(1)$ (base case: 1 element array → constant time)
  - ➩ $T(N) = 2T(N/2) + N$

Solving the Mergesort Recurrence Relation

- ✦ Can solve the recurrence by expanding the terms:

  $T(N) = 2*T(N/2) + N$
  - ➩ Since $T(N/2) = 2*T(N/4) + N/2$,
  - ➩ $T(N) = 2*[2*T(N/4) + N/2] + N$

$$
\begin{aligned}
&= 2^2*T(N/2^2) + 2*N \\
&= 2^2[2*T(N/8) + N/4] + 2*N \\
&= 2^3*T(N/2^3) + 3*N \\
&\ldots \qquad\qquad \text{(recall that } 2^{\log N} = N) \\
&= 2^{\log N}*T(N/2^{\log N}) + (\log N)*N \\
&= N * T(1) + N \log N \\
&= N * O(1) + N \log N = O(N \log N)
\end{aligned}
$$

  - ➩ $\underline{T(N) = O(N \log N)}$

Quicksort

- Mergesort requires temporary array for merging → O(N) extra space – can we do in place sorting without extra space?
- Quicksort also uses a divide and conquer strategy, but does not use the O(N) extra space
- Main Idea:
  - Partition array into left and right sub-arrays
    - Elements in left sub-array < elements in right sub-array
  - Recursively sort left and right sub-arrays
  - Concatenate left and right sub-arrays → O(1) time operation

Partitioning in Quicksort

- Choose an element from the array as the pivot
- Move all elements < pivot into left sub-array and all elements > pivot into right sub-array
  - The case where element = pivot can be handled in several ways

    7 18 2 15 9 11

  - Suppose pivot = 7
  - Left subarray = 2 Right sub-array = 18 15 9 11
- What is the running time for an array of N elements?

Quicksort Example

- Sort the array containing the elements:
  - 9 12 4 15 2 5 17 1

Questions to be Answered…

- How can we do partitioning in place?
- How do we pick the pivot to speed up running time?
- What is the best and worst case running time of Quicksort?

Answers? Next class – same place, same time…

To Do:

Read Chapter 7