Application: Geometric Hashing

Model-based object recognition is the area of computer vision that tries to recognize and locate known objects in images. A geometric model of an object is a precise model of the kind produced by CAD systems that specifies the full geometry of the object in terms of the points, lines, surfaces that define it.

Let M be an ordered set of points in a plane that constitutes the 2D model of an object. Select 3 noncollinear points of M \(e_{00}, e_{10}, e_{11}\) to be an affine basis set that defines a coordinate system on the object.

Then any point \(x\) in M can be represented in affine coordinates \((\xi, \eta)\) where

\[ x = \xi (e_{10} - e_{00}) + \eta (e_{01} - e_{00}) + e_{00} \]

Affine transforms include:
- Translation
- Rotation
- Scaling
- Skewing

So if I select a 3-point basis \(E\) for object M \(E = (e_{00}, e_{01}, e_{10})\) and I transform M by transformation \(T\) to \(M'\) and also transform my basis \(E\) to \(TE\)

\[ TE = (Te_{00}, Te_{01}, Te_{10}) \]

then if the affine coordinates of point \(x\) of M are \((\xi, \eta)\), the affine coordinates of corresponding point \(Tx\) of \(M'\) are the same \((\xi, \eta)\).

We can use quantized \((\xi, \eta)\) as two-dimensional hash table indices for object recognition.

Offline Preprocessing

Let D be a (large) database of models.
Let H be an initially empty hash table.

procedure GH_Preprocessing(D, H)
for each model M
    Extract the feature point set FM of M;
    for each noncollinear triple E of points in FM
        for each other point \(x\) of FM
            Calculate \((\xi, \eta)\) for \(x\) with respect to the transformed basis \(Te_{00}, Te_{01}, Te_{10}\);
            Store \((M, E)\) in H in bin indexed by \((\xi, \eta)\);

Online Recognition

Let H be the hash table.
Let A be an accumulator array for voting.

procedure GH_Recognition(H, A)
    Initialize A to all zeroes;
    Extract feature points FP from image;
    for each basis triple \(F\) of FP
        for each other point \(v\)
            Calculate \((\xi, \eta)\) for \(v\) with respect to \(F\);
            Retrieve the list \(L\) of model-basis pairs from the hash table H at index \((\xi, \eta)\);
            for each pair \((M, E)\) of \(L\)
    for each peak \((M, E)\) in accumulator array A
        Calculate \(T\) such that \(F = TE\);
        if (verify(T, M, FP) return T;)


Complexity

Preprocessing:
For $s$ models and $n$ points in each
$$T(s,n) = s \times n \times a = O(sn)$$
where $s$ is number of points in a model

Matching:
Best Case: You try one triple and it works.
Worst Case: You try every possible triple on the image and none of them work.

Best Case:
$$a \times n + h \times n$$
Worst Case:
$$a \times n^3 + h \times n^4$$

The worst case would be to try all bases, to get some high valued accumulators for all of them, and to try to verify all of them.