**Last Topic of Unit**

More careful look at what “two pieces of code are equivalent” means

- Fundamental software-engineering idea

- Made easier with
  - Abstraction (hiding things)
  - Fewer side effects

Not about any “new ways to code something up”
Equivalence

Must reason about “are these equivalent” all the time
   - The more precisely you think about it the better

• Code maintenance: Can I simplify this code?

• Backward compatibility: Can I add new features without changing how any old features work?

• Optimization: Can I make this code faster?

• Abstraction: Can an external client tell I made this change?

To focus discussion: When can we say two functions are equivalent, even without looking at all calls to them?
   - May not know all the calls (e.g., we are editing a library)
A definition

Two functions are equivalent if they have the same “observable behavior” no matter how they are used anywhere in any program.

Given equivalent arguments, they:

- Produce equivalent results
- Have the same (non-)termination behavior
- Mutate (non-local) memory in the same way
- Do the same input/output
- Raise the same exceptions

Notice it is much easier to be equivalent if:
- There are fewer possible arguments, e.g., with a type system and abstraction
- We avoid side-effects: mutation, input/output, and exceptions
Example

Since looking up variables in ML has no side effects, these two functions are equivalent:

\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{val } y &= 2 \\
\text{fun } f \ x &= y \times x
\end{align*}
\]

But these next two are not equivalent in general: it depends on what is passed for \( f \)

- Are equivalent if argument for \( f \) has no side-effects

\[
\begin{align*}
\text{fun } g (f, x) &= (f \ x) + (f \ x) \\
\text{val } y &= 2 \\
\text{fun } g (f, x) &= y \times (f \ x)
\end{align*}
\]

- Example: \( g ((\text{fn } i \Rightarrow \text{print } \text{"hi" } ; i), 7) \)
- Great reason for “pure” functional programming
Another example

These are equivalent \textit{only if} functions bound to \texttt{g} and \texttt{h} do not raise exceptions or have side effects (printing, updating state, etc.)

- Again: pure functions make more things equivalent

```
fun \texttt{f} \texttt{x} =
  let
    val \texttt{y} = \texttt{g} \texttt{x}
    val \texttt{z} = \texttt{h} \texttt{x}
  in
    (\texttt{y}, \texttt{z})
  end

\neq

fun \texttt{f} \texttt{x} =
  let
    val \texttt{z} = \texttt{h} \texttt{x}
    val \texttt{y} = \texttt{g} \texttt{x}
  in
    (\texttt{y}, \texttt{z})
  end
```

- Example: \texttt{g} divides by 0 and \texttt{h} mutates a top-level reference
- Example: \texttt{g} writes to a reference that \texttt{h} reads from
Syntactic sugar

Using or not using syntactic sugar is always equivalent
– By definition, else not syntactic sugar

Example:

\[
\begin{align*}
\text{fun } f \ x &= x \text{ andalso } g \ x \\
\text{fun } f \ x &= \text{if } x \text{ then } g \ x \text{ else false}
\end{align*}
\]

But be careful about evaluation order

\[
\begin{align*}
\text{fun } f \ x &= x \text{ andalso } g \ x \\
\text{fun } f \ x &= \text{if } g \ x \text{ then } x \text{ else false}
\end{align*}
\]
Standard equivalences

Three general equivalences that always work for functions
- In any (?) decent language

1. Consistently rename bound variables and uses

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ z &= z+y+z
\end{align*}
\]

But notice you can’t use a variable name already used in the function body to refer to something else

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ y &= y+y+y
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ x &= \\
\text{let } \text{val } y &= 3 \\
\text{in } x+y \text{ end}
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ y &= \\
\text{let } \text{val } y &= 3 \\
\text{in } y+y \text{ end}
\end{align*}
\]
**Standard equivalences**

Three general equivalences that always work for functions
- In (any?) decent language

2. Use a helper function or do not

\[
\begin{align*}
\text{val } y & = 14 \\
\text{fun } g \ z & = (z+y+z)+z
\end{align*}
\]

\[
\begin{align*}
\text{val } y & = 14 \\
\text{fun } f \ x & = x+y+x \\
\text{fun } g \ z & = (f \ z)+z
\end{align*}
\]

But notice you need to be careful about environments

\[
\begin{align*}
\text{val } y & = 14 \\
\text{val } y & = 7 \\
\text{fun } g \ z & = (z+y+z)+z
\end{align*}
\]

\[
\begin{align*}
\text{val } y & = 14 \\
\text{fun } f \ x & = x+y+x \\
\text{val } y & = 7 \\
\text{fun } g \ z & = (f \ z)+z
\end{align*}
\]
Standard equivalences

Three general equivalences that always work for functions
- In (any?) decent language

3. Unnecessary function wrapping

\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{fun } g \ y &= f \ y
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{val } g &= f
\end{align*}
\]

But notice that if you compute the function to call and \textit{that computation} has side-effects, you have to be careful

\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{fun } h () &= (\text{print } "\text{hi}"; f) \\
\text{fun } g \ y &= (h()) \ y
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{fun } h () &= (\text{print } "\text{hi}"; f) \\
\text{val } g &= (h())
\end{align*}
\]
One more

If we ignore types, then ML let-bindings can be syntactic sugar for calling an anonymous function:

\[
\text{let val } x = e_1 \\text{ in } e_2 \text{ end} \quad \quad \quad (\text{fn } x \Rightarrow e_2) \ e_1
\]

- These both evaluate \( e_1 \) to \( v_1 \), then evaluate \( e_2 \) in an environment extended to map \( x \) to \( v_1 \)
- So \textit{exactly} the same evaluation of expressions and result

But in ML, there is a type-system difference:
- \( x \) on the left can have a polymorphic type, but not on the right
- Can always go from right to left
- If \( x \) need not be polymorphic, can go from left to right
What about performance?

According to our definition of equivalence, these two functions are equivalent, but we learned one is awful

– (Actually we studied this before pattern-matching)

```ml
fun max xs =
  case xs of
   [] => raise Empty
   | x::[] => x
   | x::xs' =>
     if x > max xs'
     then x
     else max xs'
```

```ml
fun max xs =
  case xs of
   [] => raise Empty
   | x::[] => x
   | x::xs' =>
     let
       val y = max xs'
     in
       if x > y
       then x
       else y
     end
```
Different definitions for different jobs

- **PL Equivalence (341):** given same inputs, same outputs and effects
  - Good: Lets us replace bad $\max$ with good $\max$
  - Bad: Ignores performance in the extreme

- **Asymptotic equivalence (332):** Ignore constant factors
  - Good: Focus on the algorithm and efficiency for large inputs
  - Bad: Ignores “four times faster”

- **Systems equivalence (333):** Account for constant overheads, performance tune
  - Good: Faster means different and better
  - Bad: Beware overtuning on “wrong” (e.g., small) inputs; definition does not let you “swap in a different algorithm”

Claim: Computer scientists implicitly (?) use all three every (?) day