16 points total (2 points for Questions 1 – 6; 4 points for Question 7)

You can use up to 2 late days for this assignment.

Some of these questions involve defining the same function in different styles, to get experience with the different possibilities in Haskell.

For each top-level Haskell function you define, include a type declaration. For example, your cone_volume function for Question 1 should start with:

```
cone_volume :: Double->Double->Double
```

Please turn in one file called haskell-warmup.hs that contains all your functions and type declarations.

1. Write a function cone_volume that takes a radius and height of a cone and returns its volume. Remember to include the type declaration. (Double is a built-in Haskell type representing a double-precision floating point number.) If you write this function without a type declaration and let Haskell infer the type, it will actually come up with a more general type – but we’re going to ease into Haskell’s type system and just declare the function to use Doubles. Also use the defined Haskell constant pi.

2. Write a ascending function to test whether a list of integers is in strict ascending order. For example, ascending [1,2,3], ascending [10], and ascending [] should all return True, while ascending [2,3,1] and ascending [2,2] should both return False.

3. Write a function squares that takes a list of integers, and returns a list of the squares of those integers. For example, squares [1,2,10] should evaluate to [1,4,100], while squares [] should evaluate to []. Also try your function on an infinite list, for example squares [1..] or squares [1,3..].

   For full credit your function should be written in the pointfree style, i.e. the definition should start squares = ... (no named argument). Hint: what is the type of (^2)? What does it do?

4. Write a function parallel_resistors that calculates the total resistance of a number of resistors connected in parallel. The formula for computing this is

\[
\frac{1}{r_1} + \ldots + \frac{1}{r_n}
\]

where \( r_1, \ldots r_n \) are the resistances of each resistor. For example, parallel_resistors [10.0, 10.0], representing two 10.0 Ohm resistors in parallel, should return 5.0. Hint: if you make good use of functions in the Haskell prelude (e.g. map and recip) this is a one-line definition. This version should not be pointfree.

   Don’t worry about the edge cases of zero Ohm resistors, or no resistors (i.e. an empty list as an argument to parallel_resistors). However, after you’ve written your function, try it on these cases and see what happens. Explain why you get the results that you do in a comment.

5. Write a function pointfree_parallel_resistors that is a pointfree version of the parallel_resistors function from Question 4. Hint: if you’re stuck, think about how to build this up in easy steps. What’s an expression that will compute this list?

\([1/r_1, \ldots, 1/r_n]\)
Once you’ve got that, what expression will compute this?

\[
\frac{1}{r_1} + \ldots + \frac{1}{r_n}
\]

6. A palindrome is a sentence or phrase that is the same forwards and backwards, ignoring spaces, punctuation and other special characters, and upper vs. lower case. Some palindromes are “Madam, I’m Adam”, “Yreka Bakery”, and “Doc, note, I dissent, a fast never prevents a fatness. I diet on cod.” We’ll also consider digits (but not special characters like /) as part of the sentence or phrase — so that 01/02/2010 also counts as a palindrome. Write a Haskell function `palindrome` that takes a string as an argument and that returns True if the string is a palindrome and otherwise False. Hint: make use of the Haskell library for this problem; you shouldn’t need to write a recursive function at all for your solution. In particular, consider using functions such as `map`, `filter`, `reverse`, and assorted functions in the `Data.Char` module. If you use this module, include an `import Data.Char` statement in your program.

7. This last question involves polymorphic user-defined types. Define a polymorphic type `List` with two constructors `Null` and `Cons` for the empty list and for a non-empty list cell respectively. (This is a little silly, since of course Haskell has built-in lists, but we’re practicing.) Now define two functions `insert` and `delete` that operate on sorted lists. The `insert` function takes an item and a list, and inserts it into the correct place in the list, returning the new list. The `delete` function takes an item and a list, and returns a new list with that item deleted. If the item isn’t found, it just returns the original list. Remember that the lists are sorted, so for full credit make sure that `delete` doesn’t search down the list any further than necessary.

Include type declarations for both functions, making them the most general types possible. Also include a comment explaining the declarations, in particular explaining any type parameters and type constraints. (If you’re not sure what the most general type is, omit the declaration first and let Haskell infer it; then put it in your program.)

Here are a few examples, where the `=>` indicates what the expression evaluates to.

\[
\begin{align*}
\text{insert 3 Null} & \Rightarrow \text{Cons 3 Null} \\
\text{insert 3 (Cons 2 (Cons 4 Null))} & \Rightarrow \text{Cons 2 (Cons 3 (Cons 4 Null))} \\
\text{insert 3 (Cons 2 (Cons 3 (Cons 4 Null)))} & \Rightarrow \text{Cons 2 (Cons 3 (Cons 3 (Cons 4 Null)))} \\
\text{delete 4 (Cons 4 (Cons 4 Null))} & \Rightarrow \text{Cons 4 Null} \\
\text{delete 5 (Cons 4 (Cons 4 Null))} & \Rightarrow \text{Cons 4 (Cons 4 Null)} \\
\text{delete 1 (Cons 2 (Cons 3 (Cons 4 Null)))} & \Rightarrow \text{Cons 2 (Cons 3 (Cons 4 Null))}
\end{align*}
\]

Note that for the final delete expression, your `delete` function shouldn’t need to search down the list — as soon as it notices that 1 is less than 2, it stops looking.

Use the HUnit unit testing package to write test cases for each function. Include a test for an ordinary case, and also tests for edge cases. Also include a `run` function that runs all of your tests. (See the `UnitTestExample.hs` file linked from the Haskell page in the 341 website.)

For example, for the palindrome problem, include tests for a string that is a palindrome, a string that isn’t a palindrome, the empty string, and a date palindrome:

\[
\begin{align*}
p1 = \text{testCase} (\text{assertBool} "banana palindrome" (\text{palindrome} "Yo! Banana Boy!"))) \\
p2 = \text{testCase} (\text{assertBool} "carrot palindrome" (\text{not} (\text{palindrome} "Yo! Carrot Girl!"))) \\
p3 = \text{testCase} (\text{assertBool} "empty palindrome" (\text{palindrome ""})) \\
p4 = \text{testCase} (\text{assertBool} "date palindrome" (\text{palindrome} "01/02/2010"))
\end{align*}
\]
To test the \texttt{squares} function evaluated on an infinite list, use the \texttt{take} function from the Haskell Prelude to get the first several values and check them. (Otherwise \texttt{assertEqual} isn’t going to be happy if you just give it infinite lists.) For example:

\begin{verbatim}
s1 = testCase (assertEqual "infinite squares" [1,9,25,49] (take 4 (squares [1,3 ..])))
\end{verbatim}

For the tests involving floating-point numbers, as usual you should test for equality within an epsilon, rather than exact equality (which would often fail due to roundoff errors). If you would like, you can copy the following definition of a convenience function into your own program:

\begin{verbatim}
{- test whether a number is within epsilon of to another (for unit tests on Doubles, to accomodate floating point roundoff errors). Note that this doesn’t work for testing numbers that should be exactly 0 -- for that you should specify your own test with an appropriate epsilon -}
is_close x y = abs (x-y) < abs x * epsilon
  where epsilon = 1.0e-6
\end{verbatim}

As is often the case, there will probably be significantly more code for the tests than for the actual functions! Your program should be tastefully commented (i.e. put in a comment before each function definition saying what it does). Style counts! In particular, think about where you can use pattern matching and higher order functions to good effect to simplify your program; and avoid unnecessary repeated computations.