Please do not turn the page until 8:30.

Rules:

- The exam is closed-book, closed-note, except for both sides of one 8.5x11in piece of paper.
- **Please stop promptly at 10:20.**
- You can rip apart the pages, but please staple them back together before you leave.
- There are **100 points** total, distributed **unevenly** among **8** questions (many with multiple parts).
- When writing code, style matters, but do not worry much about indentation.

Advice:

- Read questions carefully. Understand a question before you start writing.
- Write down thoughts and intermediate steps so you can get partial credit.
- The questions are not necessarily in order of difficulty. **Skip around.** Make sure you get to all the problems.
- If you have questions, ask.
- Relax. You are here to learn.
1. (13 points) This problem uses these Racket struct definitions to define a form of binary tree:

(struct leaf (data) #:transparent)
(struct node (data left right) #:transparent)

In trees made using these structs, note that:

- Leaves and internal nodes hold data in them. The data might be any Racket value.
- We assume the left and right fields of a value built with node are themselves binary trees.

Write a `map` function over these binary trees as follows:

- It should take two curried arguments (this is not the Racket default), first a function and second a tree. (You can get partial credit without currying.)
- It should return a new tree of the same shape as the tree argument but with the function argument applied to each tree element.

**Solution:**

(define map
  (lambda (f) ; first two lines can also be just (define (map f)
    (lambda (tree)
      (if (leaf? tree)
        (leaf (f (leaf-data tree)))
        (node (f (node-data tree))
          ((map f) (node-left tree))
          ((map f) (node-right tree)))))))
2. (10 points) This problem considers the difference between

\[
\text{(define foo (lambda (x) (let ([y e1]) e2))) ; Code A}
\]

and

\[
\text{(define foo (let ([y e1]) (lambda (x) e2))) ; Code B}
\]

Assume \( e1 \) does not use \( x \).

(a) In roughly 2–4 English sentences, describe the general difference between “Code A” and “Code B.”

(b) Give an example \( e1 \) and \( e2 \) that make Code A and Code B not equivalent. You can provide additional code before the code above if you wish.

Solution:

(a) Code A evaluates \( e1 \) every time the function bound to \( \text{foo} \) is called (and not until this function is called). Code B evaluates \( e1 \) once when the binding for \( \text{foo} \) is initialized (and not again).

(b) Any example where \( e1 \) mutates memory, prints, raises an error, or does not terminate suffices.
3. (13 points) Write a Racket function `twice-each` that takes a stream s and returns a stream. (Remember a stream is a thunk that returns a pair where the cdr is a stream.) The stream returned should be like s except each value generated by s is repeated twice. For example, if s generates 1, 2, 3, 4, ..., then (twice-each s) generates 1, 1, 2, 2, 3, 3, 4, 4, ...

**Solution:**
We anticipate many correct (and incorrect) approaches will pass to a helper function whether to repeat a previous value or not, but this approach is the simplest:

```
(define (twice-each s)
  (lambda ()
    (let ([pr (s)])
      (cons (car pr)
        (lambda ()
          (cons (car pr)
            (twice-each (cdr pr))))))))
```
4. **(15 points)** In languages with first-class functions and dynamic scope, we evaluate a function in the environment where it is called (extended to map the function’s parameter name to the result of evaluating the function’s argument at the call-site). Dynamic scope is a really bad idea, but it is not difficult to implement. It is easier to implement than lexical scope because we do not need closures: we can treat functions as values.

Below is a small language definition and (partial) interpreter for DUPL, which is like MUPL from our homework except:

- The language is smaller.
- Functions are not recursive: they are like `(lambda (x) e)` in Racket or `fn x => e` in ML.
- Function calls use dynamic scope.

```
(struct var (string) #:transparent)
(struct int (num) #:transparent)
(struct add (e1 e2) #:transparent)
(struct mylet (var e1 e2) #:transparent)
(struct fun (var body) #:transparent) ; var a Racket string, body a dupl expression
(struct call (e1 e2) #:transparent)
```

```
define (envlookup env str)
  (cond [(null? env) (error "unbound variable during evaluation" str)]
        [(equal? (car (car env)) str) (cdr (car env))]
        [#t (envlookup (cdr env) str)])
(define (eval-dyn e env)
  (define (eval-dyn env str)
    (error "unbound variable during evaluation" str))
  (cond [(var? e) (envlookup env (var-string e))]
       [(int? e) e]
       [(fun? e) e]
       [(add? e) (let ((v1 (eval-dyn (add-e1 e) env))
                        (v2 (eval-dyn (add-e2 e) env)))
                   (if (and (int? v1) (int? v2))
                       (int (+ (int-num v1) (int-num v2)))
                       (error "non-int in addition")))]
       [(mylet? e) (eval-dyn (mylet-e2 e)
                              (cons (cons (mylet-var e)
                                         (eval-dyn (mylet-e1 e) env))
                                   env))]
       [(call? e) ; your answer to part (a) would go here]
       ))
```

(a) Complete the interpreter by writing the case for `call`. It should raise an error if the first argument does not evaluate to a function. Else it should call the function with the result of evaluating the argument. Use dynamic scope as described above.

(b) Complete this `DUPL` program such that:

- Evaluating it in an empty environment would produce `(int 17)`.  
- If we had lexical scope instead, then evaluation in an empty environment would cause an undefined-variable error.

```
(mylet "f" __________
  (mylet "x" __________
   (call __________ __________))
```

*The next page has room for your solutions.*
Solution:

(a) 

\[
\begin{align*}
(c\text{all}\?\;e) \\
(\text{let} \; ([v1 \; (\text{eval-dyn} \; (\text{call-e1} \; e) \; \text{env})] \\
[v2 \; (\text{eval-dyn} \; (\text{call-e2} \; e) \; \text{env})]) \\
(\text{if} \; (\text{fun?} \; v1) \\
(\text{eval-dyn} \; (\text{fun-body} \; v1) \; (\text{cons} \; (\text{cons} \; (\text{fun-var} \; v1) \; v2) \; \text{env})) \\
(\text{error} \; "\text{call attempted with non-function}")))
\end{align*}
\]

(b) Two fine solutions (there are infinitely many correct ones)

\[
\begin{align*}
(\text{mylet} \; "f" \; (\text{fun} \; "y" \; (\text{add} \; (\text{var} \; "y") \; (\text{var} \; "x")))) \\
(\text{mylet} \; "x" \; (\text{int} \; 10) \\
(\text{call} \; (\text{var} \; "f") \; (\text{int} \; 7)))
\end{align*}
\]

\[
\begin{align*}
(\text{mylet} \; "f" \; (\text{fun} \; "y" \; (\text{var} \; "x"))) \\
(\text{mylet} \; "x" \; (\text{int} \; 17) \\
(\text{call} \; (\text{var} \; "f") \; (\text{int} \; 7)))
\end{align*}
\]
5. (12 points) In this problem, we assume the purpose of the Java type system is to prevent “method missing” errors at run-time.

(a) For this purpose, is Java’s type system sound? Explain briefly, including a definition of soundness.

(b) For this purpose, is Java’s type system complete? Explain briefly, including a definition of completeness.

(c) Suppose we change the Java type system to disallow writing the constant null. Now is this revised type system sound? Explain briefly.

(d) Suppose we change the Java type system to disallow writing the constant null. Now is this revised type system complete? Explain briefly.

Solution:

(a) Soundness means no-false-negatives — the type system prevents what it is supposed to. Java’s type system is sound — we do not allow calling a method on an object unless the object actually has such a method. (We also gave full credit for unsound if the reason given is the typing of null and the definition of soundness was correct.)

(b) Completeness means no-false-positives — the type system never rejects a program unless that program could actually do what we are trying to prevent. Java’s type system is not complete — it rejects many programs that could never do a bad thing. For example, consider this code that does not type-check assuming obj has type Object: if(3 < 2) obj.m().

(c) The type system is still sound: Making a sound type system more restrictive is always sound since we accept only fewer programs.

(d) The type system is still not complete: The example given in part (b) is still valid because it does not use null.
6. (12 points) This problem uses this Ruby code:

```ruby
class A
    def m1
        self.m2()
    end
    def m2
        puts "A-m2"
    end
end

module M
    def m3
        self.m4()
    end
end

class B < A
    def m2
        puts "B-m2"
    end
end

class C < A
    include M
    def m4
        puts "C-m4"
    end
end

class D < B
    include M
end
```

For each expression below, indicate “error” if evaluation would cause a method-missing error. If not, indicate what would be printed.

(a) B.new.m1
(b) B.new.m3
(c) C.new.m1
(d) C.new.m3
(e) D.new.m1
(f) D.new.m3

**Solution:**

(a) prints B-m2
(b) method-missing
(c) prints A-m2
(d) prints C-m4
(e) prints B-m2
(f) method-missing
7. (10 points) Recall the Ruby method `is_a?` takes a class `c` as an argument and returns `true` if and only if the receiver is an instance of `c` or a subclass of `c`. Suppose `is_a?` was not provided by Ruby. Give an implementation of `is_a?` using a recursive helper method and the provided methods `class`, `superclass`, and `==`. That is, define the `is_a?` method. You can assume the argument to `is_a?` is a class.

Solution:

```ruby
def helper(c1, c2)
  c1 == c2 || (c1.superclass != nil && helper(c1.superclass, c2))
end

def is_a? c
  helper(self.class, c)
end
```
8. (15 points) In this problem, we consider a language like in lecture containing (1) records with mutable fields, (2) higher-order functions, and (3) subtyping. Like in class, record subtyping includes width and permutation but not depth, and function subtyping allows contravariant arguments and covariant results. We also add type synonyms like in ML: the declaration type t = ... means t is equivalent to the type ....

For each of the following expressions (mostly just function calls), decide if the expression should type-check, answering “Yes” if it should type-check and “No” if it should not. If your answer is, “No,” give a possible implementation of the relevant functions so that the call would try to read a field of a record that does not exist or try to use addition on a non-number.

In your solutions, use the syntax e.f to read fields and e1.f = e2 to write fields.

```
type int_pair = { car : int, cdr : int }
type pair_pair = { car : int_pair, cdr : int_pair }
type int_triple = { car : int, cdr : int, cgr : int }
type int_pair_fn = int_pair -> int_pair

val r1 = { cgr=5, cdr=6, car=9 }
val r2 = { car = { car = 1, cdr = 2}, cdr = { car = 3, cdr = 4 } }
val r3 = { car = r1, cdr = { car = 3, cdr = 4 } }
val r4 = { car = 7, cdr = 9 } (* assume these variables are bound to functions with the given type *)
val f1 : int_pair -> int_pair = ...
val f2 : pair_pair -> int_pair = ...
val f3 : int_triple -> int_pair = ...
val f4 : { car : int } -> int_triple = ...
val f5 : int_pair_fn -> int_pair = ...
```

(a) f1(r1)
(b) f1(r2)
(c) f2(r2) followed by r2.car.cdr
(d) f2(r3) followed by r3.car.cgr
(e) f3(r1)
(f) f3(r4)
(g) f5(f1)
(h) f5(f3)
(i) f5(f4)

Solution:

(a) Yes
(b) No, val f1 = fn x => {car = x.car + 0, cdr = 0}
(c) Yes
(d) No, val f2 = fn x => ((x.car = { car = 3, cdr = 4 }); x.car)
(e) Yes
(f) No, val f3 = fn x => {car=x.cgr, cdr=0}

(g) Yes

(h) No, val f3 = fn x => {car=x.cgr, cdr=0} and val f5 = fn g => g { car = 1, cdr = 2 }

(i) Yes
Name:__________________________________________

More room for answers if needed.