Last Topic of Unit

More careful look at what “two pieces of code are equivalent” means

– Fundamental software-engineering idea

– Made easier with
  • Abstraction (hiding things)
  • Fewer side effects

Not about any “new ways to code something up”
Equivalence

Must reason about “are these equivalent” all the time
  – The more precisely you think about it the better

• **Code maintenance**: Can I simplify this code?

• **Backward compatibility**: Can I add new features without changing how any old features work?

• **Optimization**: Can I make this code faster?

• **Abstraction**: Can an external client tell I made this change?

To focus discussion: When can we say two functions are equivalent, even without looking at all calls to them?
  – May not know all the calls (e.g., we are editing a library)
A definition

Two functions are equivalent if they have the same “observable behavior” no matter how they are used anywhere in any program.

Given equivalent arguments, they:
- Produce equivalent results
- Have the same (non-)termination behavior
- Mutate (non-local) memory in the same way
- Do the same input/output
- Raise the same exceptions

Notice it is much easier to be equivalent if:
- There are fewer possible arguments, e.g., with a type system and abstraction
- We avoid side-effects: mutation, input/output, and exceptions
Example

Since looking up variables in ML has no side effects, these two functions are equivalent:

\[
\text{fun } f \ x = x + x \quad \text{and} \quad \text{val } y = 2 \text{ fun } f \ x = y \times x
\]

But these next two are not equivalent in general: it depends on what is passed for \( f \):

- Are equivalent if argument for \( f \) has no side-effects

\[
\text{fun } g \ (f,x) = (f \ x) + (f \ x) \quad \not= \quad \text{val } y = 2 \text{ fun } g \ (f,x) = y \times (f \ x)
\]

- Example: \( g \ ((\text{fn } i => \text{print } "\text{hi}" ; i), 7) \)
- Great reason for “pure” functional programming
Another example

These are equivalent only if functions bound to $g$ and $h$ do not raise exceptions or have side effects (printing, updating state, etc.)

- Again: pure functions make more things equivalent

```
fun f x =
  let
    val y = g x
    val z = h x
  in
    (y, z)
  end

fun f x =
  let
    val z = h x
    val y = g x
  in
    (y, z)
  end
```

- Example: $g$ divides by 0 and $h$ mutates a top-level reference
- Example: $g$ writes to a reference that $h$ reads from
**Syntactic sugar**

Using or not using syntactic sugar is always equivalent
- By definition, else not syntactic sugar

Example:

\[
\begin{align*}
\text{fun } f \ x &= x \ \text{andalso} \ g \ x \\
\text{fun } f \ x &= \text{if } x \ \text{then } g \ x \ \text{else } \text{false} \\
\text{fun } f \ x &= \text{if } g \ x \ \text{then } x \ \text{else } \text{false} \\
\end{align*}
\]

But be careful about evaluation order
Standard equivalences

Three general equivalences that always work for functions
   – In any (?) decent language

1. Consistently rename bound variables and uses

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ z &= z+y+z
\end{align*}
\]

But notice you can’t use a variable name already used in the
function body to refer to something else

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ y &= y+y+y
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ x &= \\
& \quad \text{let } \text{val } y = 3 \\
& \quad \text{in } x+y \ \text{end}
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ y &= \\
& \quad \text{let } \text{val } y = 3 \\
& \quad \text{in } y+y \ \text{end}
\end{align*}
\]
Standard equivalences

Three general equivalences that always work for functions
– In (any?) decent language

2. Use a helper function or do not

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } g \ z &= (z+y+z)+z
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x \\
\text{fun } g \ z &= (f \ z)+z
\end{align*}
\]

But notice you need to be careful about environments

\[
\begin{align*}
\text{val } y &= 14 \\
\text{val } y &= 7 \\
\text{fun } g \ z &= (z+y+z)+z
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x \\
\text{val } y &= 7 \\
\text{fun } g \ z &= (f \ z)+z
\end{align*}
\]
Standard equivalences

Three general equivalences that always work for functions
– In (any?) decent language

3. Unnecessary function wrapping

\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{fun } g \ y &= f \ y
\end{align*}
\]
\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{val } g &= f
\end{align*}
\]

But notice that if you compute the function to call and that computation has side-effects, you have to be careful

\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{fun } h () &= (\text{print } "\text{hi}"; f) \\
\text{fun } g y &= (h()) \ y
\end{align*}
\]
\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{fun } h () &= (\text{print } "\text{hi}"; f) \\
\text{val } g &= (h())
\end{align*}
\]
One more

If we ignore types, then ML let-bindings can be syntactic sugar for calling an anonymous function:

```
let val x = e1
  in e2 end

(fn x => e2) e1
```

- These both evaluate $e_1$ to $v_1$, then evaluate $e_2$ in an environment extended to map $x$ to $v_1$
- So *exactly* the same evaluation of expressions and result

But in ML, there is a type-system difference:
- $x$ on the left can have a polymorphic type, but not on the right
- Can always go from right to left
- If $x$ need not be polymorphic, can go from left to right
What about performance?

According to our definition of equivalence, these two functions are equivalent, but we learned one is awful

– (Actually we studied this before pattern-matching)

```plaintext
def max xs =
  case xs of
    [] => raise Empty
  | x::[] => x
  | x::xs' =>
    if x > max xs'
    then x
    else max xs'
```

```plaintext
def max xs =
  case xs of
    [] => raise Empty
  | x::[] => x
  | x::xs' =>
    let
      val y = max xs'
    in
      if x > y
      then x
      else y
    end
```
Different definitions for different jobs

- **PL Equivalence (341):** given same inputs, same outputs and effects
  - Good: Lets us replace bad $\text{max}$ with good $\text{max}$
  - Bad: Ignores performance in the extreme

- **Asymptotic equivalence (332):** Ignore constant factors
  - Good: Focus on the algorithm and efficiency for large inputs
  - Bad: Ignores “four times faster”

- **Systems equivalence (333):** Account for constant overheads, performance tune
  - Good: Faster means different and better
  - Bad: Beware overtuning on “wrong” (e.g., small) inputs; definition does not let you “swap in a different algorithm”

*Claim:* Computer scientists implicitly (?) use all three every (?) day