CSE341: Programming Languages
Lecture 10
References, Polymorphic Datatypes, the Value Restriction, Type Inference

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Callbacks

A common idiom: Library takes functions to apply later, when an event occurs – examples:

- When a key is pressed, mouse moves, data arrives
- When the program enters some state (e.g., turns in a game)

A library may accept multiple callbacks

- Different callbacks may need different private data with different types
- Fortunately, a function’s type does not include the types of bindings in its environment
- (In OOP, objects and private fields are used similarly, e.g., Java Swing’s event-listeners)
Mutable state

While it’s not absolutely necessary, mutable state is reasonably appropriate here

- We really do want the “callbacks registered” and “events that have been delivered” to change due to function calls

For the reasons we have discussed, ML variables really are immutable, but there are mutable references (use sparingly)

- New types: \( t \text{ ref} \) where \( t \) is a type
- New expressions:
  - \( \text{ref } e \) to create a reference with initial contents \( e \)
  - \( e_1 := e_2 \) to update contents
  - \( !e \) to retrieve contents (not negation)
A variable bound to a reference (e.g., x) is still immutable: it will always refer to the same reference.

But the contents of the reference may change via :=

And there may be aliases to the reference, which matter a lot

Reference are first-class values

Like a one-field mutable object, so := and ! don’t specify the field
Example call-back library

Library maintains mutable state for “what callbacks are there” and provides a function for accepting new ones

- A real library would support removing them, etc.
- In example, callbacks have type \( \text{int} \rightarrow \text{unit} \) (executed for side-effect)

So the entire public library interface would be the function for registering new callbacks:

\[
\text{val onKeyEvent : (int \rightarrow \text{unit}) \rightarrow \text{unit}}
\]
Library implementation

```ocaml
val cbs : (int -> unit) list ref = ref []

fun onKeyEvent f = cbs := f :: (!cbs)

fun onEvent i =
  let fun loop fs =
      case fs of
        []   => ()
      | f::fs' => (f i; loop fs')
  in loop (!cbs) end
```
Clients

Can only register an \texttt{int -> unit}, so if any other data is needed, must be in closure’s environment

- And if need to “remember” something, need mutable state

Examples:

```haskell
val timesPressed = ref 0
val _ = onKeyEvent (fn _ =>
    timesPressed := (!timesPressed) + 1)

fun printIfPressed i =
    onKeyEvent (fn j =>
        if i=j
        then print ("pressed " ^ Int.toString i)
        else ()
    )
```
More about types

• Polymorphic datatypes, type constructors
• Why do we need the Value Restriction?
• Type inference: behind the curtain
Polymorphic Datatypes

datatype int_list =
    EmptyList
  | Cons of int * int_list

datatype 'a non_mt_list =
    One of 'a
  | More of 'a * ('a non_mt_list)

datatype ('a,'b) tree =
    Leaf of 'a
  | Node of 'b * ('a,'b) tree * ('a,'b) tree

val t1 = Node("hi",Leaf 4,Leaf 8)
        (* (int,string) tree *)
val t2 = Node("hi",Leaf true,Leaf 8)
        (* does not typecheck *)
Polymorphic Datatypes

```haskell
datatype 'a list = [] | :: of 'a * ('a list)
(* if this were valid syntax *)

datatype 'a option = NONE | SOME of 'a
```

- list, tree, etc. are not types; they are type constructors.
- int list, (string, real) tree, etc. are types.
- Pattern-matching works on all datatypes.
The Value Restriction Appears 😞

If you use partial application to create a polymorphic function, it may not work due to the value restriction

- Warning about “type vars not generalized”
  - And won’t let you call the function

- This should surprise you; you did nothing wrong 😊 but you still must change your code

- See the written lecture summary about how to work around this wart (and ignore the issue until it arises)

- The wart is there for good reasons, related to mutation and not breaking the type system
Purpose of the Value Restriction

val xs = ref []
  (* xs : 'a list ref *)
val _ = xs := ["hi"]
  (* instantiate 'a with string *)
val y = 1 + (hd (!xs))
  (* BAD: instantiate 'a with int *)

• A binding is only allowed to be polymorphic if the right-hand side is:
  – a variable; or
  – a value (including function definitions, constructors, etc.)
• ref [] is not a value, so we can only give it non-polymorphic types such as int list ref or string list ref, but not 'a list ref.
Downside of the Value Restriction

\[
\text{val } \text{pr\_list} = \text{List.map (fn x => (x,x)) (* X *)}
\]

\[
\text{val } \text{pr\_list} : \text{int list} \to (\text{int*int}) \text{ list} = \\
\text{List.map (fn x => (x,x))}
\]

\[
\text{val } \text{pr\_list} = \\
\text{fn lst} \Rightarrow \text{List.map (fn x => (x,x)) lst}
\]

\[
\text{fun } \text{pr\_list} \text{ lst} = \text{List.map (fn x => (x,x)) lst}
\]

- The SML type checker does not know if the ‘a list type uses references internally, so it has to be conservative and assume it could.
- In practice, this means we need to be more explicit about partial application of polymorphic functions.
Type inference: sum

fun sum xs =
   case xs of
      [] => 0
      | x::xs' => x + (sum xs')

sum : t1 -> t2
xs : t1
Type inference: sum

fun sum xs =
  case xs of
     [] => 0
   | x::xs' => x + (sum xs')

sum : t1 -> t2
xs : t1

t1 = t5  list
Type inference: sum

fun sum xs =
  case xs of
    [] => 0
  | x::xs' => x + (sum xs')

sum : t1 -> t2
xs : t1

t1 = t5 list

t2 = int
Type inference: sum

fun sum xs =
case xs of
  [] => 0
  | x::xs' => x + (sum xs')

sum : t1 -> t2
xs : t1
x : t3
t1 = t5 list
t2 = int
Type inference: sum

fun sum xs =
  case xs of
    [] => 0
  | x :: xs' => x + (sum xs')

sum : t1 -> t2
xs : t1
x : t3
xs' : t4

t1 = t5 list
t2 = int
Type inference: sum

fun sum xs =
    case xs of
        [] => 0
    | x :: xs' => x + (sum xs')

sum : t1 -> t2
xs : t1
x : t3
xs' : t4
t1 = t5 list

t2 = int


t3 = t5
Type inference: sum

fun sum xs = 
case xs of 
  [] => 0 
  | x::xs' => x + (sum xs')

sum : t1 -> t2 
x : t3
xs : t1
xs' : t4
t1 = t5 list
t2 = int
t3 = t5
t4 = t5 list
Type inference: sum

```plaintext
fun sum xs =
  case xs of
    [] => 0
  | x::xs' => x + (sum xs')

sum : t1 -> t2
xs : t1
tax : t3
xs' : t4
```

t1 = t5  list

t2 = int

t3 = t5

t4 = t5  list

t3 = int
Type inference: sum

```fun
sum xs =
    case xs of
        [] => 0
    | x::xs' => x + (sum xs')
```

\[
\begin{align*}
\text{sum} &: t1 \to t2 \\
x : # & t1 \\
x &: t3 \\
x$: & t4 \\
t1 &= t5 \text{ list} \\
t2 &= \text{int} \\
t3 &= t5 \\
t4 &= t5 \text{ list} \\
t3 &= \text{int} \\
t1 &= t4
\end{align*}
\]
Type inference: sum

\[
\text{fun} \ \text{sum} \ \text{xs} = \\
\text{case} \ \text{xs} \ \text{of} \\
\quad [] \Rightarrow 0 \\
\quad \text{x::xs'} \Rightarrow \text{x} + (\text{sum} \ \text{xs'})
\]

\[
\begin{align*}
\text{sum} & : \ t1 \rightarrow \ t2 \\
\text{xs} & : \ t1 \\
\text{x} & : \ t3 \\
\text{xs'} & : \ t1 \\
\text{t1} & = \ t5 \ \text{list} \\
\text{t2} & = \ \text{int} \\
\text{t3} & = \ t5 \\
\text{t4} & = \ t5 \ \text{list} \\
\text{t3} & = \ \text{int} \\
\text{t1} & = \ t4
\end{align*}
\]
Type inference: sum

```haskell
fun sum xs =
    case xs of
        [] => 0
    | x::xs' => x + (sum xs')
```

sum : t1 -> t2
xs : t1
x : t3
xs' : t1

\[
\begin{align*}
t1 &= t5 \quad \text{list} \\
t2 &= \text{int} \\
t3 &= t5 \\
t4 &= t5 \quad \text{list} \\
t1 &= t4
\end{align*}
\]
Type inference: sum

\[
\text{fun sum xs =}
\begin{cases}
    \text{case xs of} & \\
    [] & \Rightarrow 0 & \\
    x :: xs' & \Rightarrow x + (\text{sum xs'})
\end{cases}
\]

\[
\text{sum : t1 } \rightarrow \text{ t2} \\
\text{xs : t1} \\
\text{x : t3} \\
\text{xs' : t1}
\]

\[
\text{t1 = t5 list} \\
\text{t2 = int} \\
\text{t3 = t5} \\
\text{t1 = t5 list} \\
\text{t3 = int} \\
\text{t1 = t4}
\]
Type inference: sum

fun sum xs =
    case xs of
        [] => 0
    | x::xs' => x + (sum xs')

sum : t1 -> t2
xs : t1
x : t3
xs' : t1

t1 = t5  list
t2 = int
t3 = t5
t1 = t5  list
t3 = int
t1 = t4
Type inference: sum

fun sum xs =
    case xs of
        [] => 0
    | x::xs' => x + (sum xs')

sum : t1 -> t2
xs : t1
    x : int
xs' : t1

t1 = t5  list
t2 = int
t3 = t5
t4 = t5  list
t5 = int
t1 = t4
Type inference: sum

fun sum xs =
    case xs of
        [] => 0
    | x::xs' => x + (sum xs')

sum : t1 -> t2
xs : t1
    x : int
xs' : t1
Type inference: sum

```haskell
fun sum xs =
  case xs of
    [] => 0
    | x :: xs' => x + (sum xs')
```

sum : t1 -> t2
xs : t1
x : int
xs' : t1

t1 = t5 list
int = t5
t2 = int
int = t5
t3 = int
t1 = t4
t1 = t4
Type inference: sum

fun sum xs = 
  case xs of 
    [] => 0 
  | x::xs' => x + (sum xs')

sum : t1 -> t2 
xs : t1 
  x : int
xs' : t1 

\[ t1 = \text{int list} \]
\[ t2 = \text{int} \]
\[ \text{int} = t5 \]
\[ t1 = t5 \text{ list} \]
\[ t3 = \text{int} \]
\[ t1 = t4 \]
Type inference: sum

fun sum xs =  
    case xs of  
        [] => 0  
    | x::xs' => x + (sum xs')

sum : t1 -> t2  
xs : t1  
  x : int  
xs' : t1

t1 = int list  
t2 = int  
int = t5  
t1 = t5 list  
t3 = int  
t1 = t4
Type inference: sum

fun sum xs =
    case xs of
        [] => 0
    | x::xs' => x + (sum xs')

sum : t1 -> int
xs : t1
    x : int
xs' : t1
t1 = int list
  t2 = int
  int = t5
  t1 = t5 list
  t3 = int
  t1 = t4
Type inference: sum

fun sum xs =
    case xs of
        [] => 0
    | x::xs' => x + (sum xs')

sum : t1 -> int
xs : t1
x : int
xs' : t1
t1 = int list
t2 = int
int = t5
t1 = t5 list
t3 = int
t1 = t4
Type inference: sum

fun sum xs =
  case xs of
      [] => 0
    | x::xs' => x + (sum xs')

sum : int list -> int
xs : int list
x : int
xs' : int list
t1 = int list
t2 = int
int = t5
t1 = t5 list
t3 = int
t1 = t4
Type inference: length

fun length xs =
  case xs of
  [] => 0
  | _ :: xs' => 1 + (length xs')

length : t1 -> t2
xs : t1
Type inference: length

fun length xs =
  case xs of
    [] => 0
  | _ :: xs' => 1 + (length xs')

length : t1 -> t2
xs : t1

t1 = t4 list
Type inference: length

fun length xs =
  case xs of
    [] => 0
    _ :: xs' => 1 + (length xs')

length : t1 -> t2
xs : t1
t1 = t4 list
t2 = int
Type inference: length

```
fun length xs =
  case xs of
    [] => 0
  | _ :: xs' => 1 + (length xs')
```

\[ length : \text{t1} \rightarrow \text{t2} \]
\[ \text{xs} : \text{t1} \]
\[ \text{xs}^' : \text{t3} \]

\( \text{t1} = \text{t4 list} \)
\( \text{t2} = \text{int} \)
Type inference: length

```haskell
fun length xs =
  case xs of
    [] => 0
    _ :: xs' => 1 + (length xs')
```

length : t1 -> t2
xs : t1
xs' : t3
t1 = t4 list
t2 = int
t3 = t4 list
Type inference: length

fun length xs =
    case xs of
        [] => 0
    | _::xs' => 1 + (length xs')

length : t1 -> t2
     xs : t1
    xs' : t3

t1 = t4 list
t2 = int
t3 = t4 list
t1 = t3
Type inference: length

```haskell
fun length xs = 
  case xs of
    [] => 0
    _ :: xs' => 1 + (length xs')
```

```
length : t1 -> t2
  xs : t1
  xs' : t1

  t1 = t4 list
t2 = int
t1 = t4 list
t1 = t3
```
Type inference: length

\[
\text{fun } \text{length } \text{xs } = \\
\quad \text{case } \text{xs } \text{of} \\
\quad \quad [] \Rightarrow 0 \\
\quad | \ _::\text{xs'} \Rightarrow 1 + (\text{length } \text{xs'})
\]

\[
\text{length : } t1 \rightarrow t2 \\
\quad \text{xs : } t1 \\
\quad \text{xs'} : t1 \\
\text{t1 } = t4 \text{ list} \\
\text{t2 } = \text{int} \\
\text{t1 } = t4 \text{ list} \\
\text{t1 } = t3
\]
Type inference: length

fun length xs =
  case xs of
    []  =>  0
    _ :: xs' =>  1 + (length xs')

length : t1 -> t2
xs : t1
xs' : t1

 t1 = t4 list
 t2 = int
 t1 = t4 list
 t1 = t3
Type inference: length

fun length xs =
    case xs of
        [] => 0
    | _ :: xs' => 1 + (length xs')

length : t1 -> int
xs : t1
xs' : t1
t1 = t4 list
t2 = int
t1 = t4 list
t1 = t3
Type inference: length

fun length xs =
    case xs of
      [] => 0
    | _ :: xs' => 1 + (length xs')

length : t1 -> int
xs : t1
xs' : t1

\[ t1 = t4 \text{ list} \]
\[ t2 = \text{int} \]
\[ t1 = t4 \text{ list} \]
\[ t1 = t3 \]
Type inference: length

fun length xs =
  case xs of
  [] => 0
  | _ :: xs' => 1 + (length xs')

length : t4 list -> int
xs : t4 list -> int
xs' : t4 list

t1 = t4 list

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Type inference: length

```haskell
fun length xs =
  case xs of
    [] => 0
  | _::xs' => 1 + (length xs')
```

length : `a list -> int
x : t4 list -> int
xs' : t4 list

length works no matter what `a is.
Type inference: compose

fun compose (f,g) = fn x => f (g x)

compose : t1 * t2 -> t3
  f : t1
  g : t2
Type inference: compose

fun compose (f,g) = fn x => f (g x)

compose : t1 * t2 -> t3
    f : t1
    g : t2
        t3 = t4 -> t5
Type inference: compose

fun compose (f,g) = fn x => f (g x)

compose : t1 * t2 -> t3
f : t1
g : t2
tax : t4
t3 = t4 -> t5
Type inference: compose

fun compose (f, g) = fn x => f (g x)

compose : t1 * t2 -> t3
f : t1
f : t1

g : t2
          t3 = t4 -> t5
x : t4
          t2 = t4 -> t6
Type inference: compose

fun compose (f,g) = fn x => f (g x)

compose : t1 * t2 -> t3

f : t1

g : t2

x : t4

t3 = t4 -> t5

t2 = t4 -> t6

t1 = t6 -> t7
fun compose (f,g) = fn x => f (g x)

compose : t1 * t2 -> t3
  f : t1
  g : t2
  x : t4

  t3 = t4 -> t5
  t2 = t4 -> t6
  t1 = t6 -> t7
  t5 = t7
Type inference: compose

fun compose (f,g) = fn x => f (g x)

compose : t1 * t2 -> t3
  f : t1
  g : t2
  x : t4
t3 = t4 -> t5
t2 = t4 -> t6
t1 = t6 -> t5
t5 = t7
Type inference: compose

fun compose (f,g) = fn x => f (g x)

compose : t1 * t2 -> t3
  f : t1
  g : t2
  x : t4

  t1 = t6 -> t5
  t2 = t4 -> t6
  t3 = t4 -> t5
  t5 = t7
fun compose (f,g) = fn x => f (g x)

compose : (t6 -> t5) * t2 -> t3
f : t6 -> t5
  t3 = t4 -> t5
g : t2
  t2 = t4 -> t6
x : t4
  t1 = t6 -> t5
t5 = t7
Type inference: compose

fun compose (f,g) = fn x => f (g x)

compose : (t6 -> t5) * t2 -> t3
f : t6 -> t5

g : t2

x : t4

f3 = t4 -> t5
t2 = t4 -> t6
t1 = t6 -> t5
t5 = t7
Type inference: compose

\[
\text{fun compose } (f,g) = \text{fn } x \Rightarrow f \ (g \ x)
\]

\[
\text{compose : } (t6 \rightarrow t5) \times (t4 \rightarrow t6) \rightarrow t3
\]

\[
f : t6 \rightarrow t5
\]

\[
g : t4 \rightarrow t6
\]

\[
x : t4
\]

\[
t3 = t4 \rightarrow t5
\]

\[
t2 = t4 \rightarrow t6
\]

\[
t1 = t6 \rightarrow t5
\]

\[
t5 = t7
\]
Type inference: compose

\[
\text{fun compose (f, g) = fn } x \Rightarrow f (g \ x)\]

\[
\text{compose : } (t6 \rightarrow t5) \times (t4 \rightarrow t6) \rightarrow t3
\]

\[
f : t6 \rightarrow t5
\]

\[
g : t4 \rightarrow t6
\]

\[
x : t4
\]

\[
t3 = t4 \rightarrow t5
\]

\[
t2 = t4 \rightarrow t6
\]

\[
t1 = t6 \rightarrow t5
\]

\[
t5 = t7
\]
Type inference: compose

```plaintext
fun compose (f, g) = fn x => f (g x)
```

```
compose  :  (t6 -> t5) * (t4 -> t6) -> (t4 -> t5)
f        :  t6 -> t5
g        :  t4 -> t6
x        :  t4 -> t5
```

```
t3 = t4 -> t5
t2 = t4 -> t6
t1 = t6 -> t5
t5 = t7
```
Type inference: compose

\[
\text{fun } \text{compose} \ (f, g) = \text{fn } x \Rightarrow f \ (g \ x)
\]

\[
\text{compose} : \ (t6 \to t5) \times (t4 \to t6) \to (t4 \to t5)
\]

\[
f : t6 \to t5
\]

\[
g : t4 \to t6
\]

\[
x : t4 \to t5
\]

\[
t3 = t4 \to t5
\]

\[
t2 = t4 \to t6
\]

\[
t1 = t6 \to t5
\]

\[
t5 = t7
\]
fun compose (f,g) = fn x => f (g x)

compose : ('a -> 'b) * ('c -> 'a) -> ('c -> 'b)

f : t6 -> t5

g : t4 -> t6

x : t4 -> t5

t3 = t4 -> t5

t2 = t4 -> t6

t1 = t6 -> t5

t5 = t7
fun compose (f,g) = fn x => f (g x)

compose : ('a -> 'b) * ('c -> 'a) -> ('c -> 'b)

f : t6 -> t5

g : t4 -> t6

x : t4 -> t5

t3 = t4 -> t5

t2 = t4 -> t6

t1 = t6 -> t5

t5 = t7

compose : ('b -> 'c) * ('a -> 'b) -> ('a -> 'c)
Type inference: broken sum

fun sum xs =
  case xs of
    [] => 0
    | x::xs' => x + (sum x)

sum : t1 -> t2
xs : t1
Type inference: broken sum

```haskell
fun sum xs =
  case xs of
    [] => 0
    | x::xs' => x + (sum x)
```

```
sum : t1 -> t2
xs : t1
```

t1 = t5 list
Type inference: broken sum

fun sum xs =  
  case xs of  
   [] => 0  
   | x::xs' => x + (sum x)

sum : t1 -> t2  
xss : t1  
t1 = t5 list  
t2 = int
Type inference: broken sum

fun sum xs =
  case xs of
    [] => 0
  | x :: xs' => x + (sum x)

sum : t1 -> t2  t1 = t5 list
xs : t1                  t2 = int
x : t3
Type inference: broken sum

\[
\text{fun sum xs = case xs of}\n\begin{align*}
[&] & \Rightarrow 0 \\
| \ x::xs' & \Rightarrow x + (\text{sum } x)
\end{align*}
\]

sum : t1 -> t2 \quad t1 = t5 \quad \text{list}
xs : t1 \quad t2 = \text{int}

x : t3
xs' : t4
Type inference: broken sum

```haskell
fun sum xs = 
case xs of
    [] => 0
    x::xs' => x + (sum x)
```

\[
\begin{align*}
sum &: \text{t}_1 \to \text{t}_2 \\
x_1 &: \text{t}_1 \\
x &: \text{t}_3 \\
x_2 &: \text{t}_4
\end{align*}
\]

\[
\begin{align*}
t_1 &= \text{t}_5 \ \text{list} \\
t_2 &= \text{int} \\
t_3 &= \text{t}_5 \\
t_4 &= \text{t}_5
\end{align*}
\]
Type inference: broken sum

```plaintext
fun sum xs = 
    case xs of
        [] => 0
    | x::xs' => x + (sum x)
```

```
sum : t1 -> t2  t1 = t5 list
xs : t1       t2 = int
  x : t3       t3 = t5
xs' : t4      t4 = t5 list
```
Type inference: broken sum

```haskell
fun sum xs =
  case xs of
    [] => 0
  | x::xs' => x + (sum x)
```

\[\text{sum : } t_1 \to t_2\]
\[\text{xs : } t_1\]
\[\text{x : } t_3\]
\[\text{xs' : } t_4\]
\[t_1 = t_5 \text{ list}\]
\[t_2 = \text{int}\]
\[t_3 = t_5\]
\[t_4 = t_5 \text{ list}\]
\[t_3 = \text{int}\]
Type inference: broken sum

fun sum xs = 
case xs of 
  [] => 0 
  | x::xs' => x + (sum x)

sum : t1 -> t2 
xs : t1 
x : t3 
xs' : t4

t1 = t5 list 
t2 = int 
t3 = t5 
t4 = t5 list 
t3 = int 
t1 = t3
Type inference: sum

\[
\text{fun } \text{sum } \text{xs } = \\
\text{case } \text{xs } \text{of } \\
\quad [] \Rightarrow 0 \\
\quad x::xs' \Rightarrow x + (\text{sum } x)
\]

\[
\text{sum} : t1 \rightarrow t2 \\
x \text{ : } t1 \\
x \text{ : } t1 \\
x \text{s'} : t4 \\
t1 = t5 \text{ list} \\
t2 = \text{int} \\
t1 = t5 \\
t4 = t5 \text{ list} \\
t1 = \text{int} \\
t1 = t3
\]
Type inference: sum

fun sum xs = 
case xs of 
  [] => 0
| x::xs' => x + (sum x)

sum : t1 -> t2
xs : t1
x : t1
xs' : t4

t1 = t5  list

xs = int

t1 = t5

x = t5

xs' = t5  list

t1 = int

xs' = t3
Type inference: sum

fun sum xs =  
  case xs of  
    [] => 0  
  | x::xs' => x + (sum x)

sum : int-> t2
xs : int
  int = t5  list
x : int
  t2 = int
xs' : t4
  int = t5
  t4 = t5  list
  t1 = int
  t1 = t3
Type inference: sum

```
fun sum xs =
  case xs of
    [] => 0
  | x::xs' => x + (sum x)
```

```
sum : int -> t2
xs : int
x : int
xs' : t4
```

```
int = t5 list
t2 = int
int = t5
t4 = t5 list
t1 = int
t1 = t3
```
Type inference: sum

```plaintext
fun sum xs =
  case xs of
    [] => 0
  | x::xs' => x + (sum x)
```

sum : \texttt{int} -> \texttt{t2}
xs : \texttt{int}
\begin{align*}
x : \texttt{int} \\
xss' : \texttt{t5 list}
\end{align*}

\begin{align*}
\texttt{int} &= \texttt{t5 list} \\
\texttt{t2} &= \texttt{int} \\
\texttt{int} &= \texttt{t5} \\
\texttt{t4} &= \texttt{t5 list} \\
\texttt{t1} &= \texttt{int} \\
\texttt{t1} &= \texttt{t3}
\end{align*}
Type inference: sum

```haskell
fun sum xs =
  case xs of
    [] => 0
  | x :: xs' => x + (sum x)
```

```plaintext
sum : int -> t2
xs : int
x : int
xs' : t5 list
```

```
int = t5 list
int = t5
int = t5
int = t5
```

```plaintext
ten = t3
```
Type inference: sum

```haskell
fun sum xs =
  case xs of
    [] => 0
    x::xs' => x + (sum x)
```

sum : int-> t2
xs : int
  x : int
xs' : int list

int = int list
  t2 = int
int = t5
  t4 = t5 list
  t1 = int
  t1 = t3
Type inference: sum

\[
\text{fun sum xs =}
\]
\[
\text{case xs of}
\]
\[
\text{[]} => 0
\]
\[
\text{\textbar \ x::xs'} => x + (\text{sum x})
\]

\[
\text{sum : int-> t2}
\]
\[
\text{xs : int}
\]
\[
\text{x : int}
\]
\[
\text{xs' : int list}
\]

\[
\text{int = int list}
\]
\[
\text{t2 = int}
\]
\[
\text{int = t5}
\]
\[
\text{t4 = t5 list}
\]
\[
\text{t1 = int}
\]
\[
\text{t1 = t3}
\]
Type inference: sum

\[
\text{fun } \text{sum } \text{xs} = \\
\quad \text{case } \text{xs} \text{ of} \\
\quad \quad [ ] \Rightarrow 0 \\
\quad \quad x :: x's \Rightarrow x + (\text{sum } x)
\]

\[\text{sum : int} \rightarrow \text{int} \]
\[\text{xs : int} \]
\[\text{x : int} \]
\[\text{xs' : int list} \]

\[\text{int} = \text{int list} \]
\[\text{t2 = int} \]
\[\text{int} = \text{t5} \]
\[\text{t4 = t5 list} \]
\[\text{t1 = int} \]
\[\text{t1 = t3} \]
Type inference: sum

fun sum xs =
  case xs of
    [] => 0
    | x::xs' => x + (sum x)

sum : int-> int
xs : int
  x : int
xs' : int list

int = int list
t2 = int
int = t5
t4 = t5 list
t1 = int
t1 = t3
Parting comments on ML type inference

• You almost never have to write types in ML (even on parameters), with some minor caveats.
• Hindley-Milner type inference algorithm
• ML has no subtyping. If it did, the equality constraints we used for inference would be overly restrictive.
• Type variables and inference are not tied to each. Some languages have one without the other.
  – Type variables alone allow convenient code reuse.
  – Without type variables, we cannot give a type to compose until we see it used.