You will define several SML functions. Many will be very short because they will use other higher-order functions. The latter part of the assignment relates to implementing pattern-matching somewhat like in ML. You may use functions in ML’s library; the problems point you toward the useful functions. The sample solution is about 120 lines, including the provided code. This assignment is probably more difficult than homework 2 even though (perhaps because) many of the problems have 1-line answers.

Download hw3provided.sml from the course website.

1. Write a function only_capitals that takes a string list and returns a string list that has only the strings in the argument that start with an uppercase letter. Assume all strings have at least 1 character. Use library functions List.filter, Char.isUpper, and String.sub to make a 1-2 line solution.

2. Write a function longest_string1 that takes a string list and returns the longest string in the list. If the list is empty, return "". In the case of a tie, return the string closest to the beginning of the list. Use foldl, String.size, and no (other) recursion.

3. Write a function longest_string2 that is exactly like longest_string1 except in the case of ties it returns the string closest to the end of the list. Your solution should be almost an exact copy of longest_string1.

4. Write functions longest_string_helper, longest_string3, and longest_string4 such that:
   - longest_string3 has the same behavior as longest_string1 and longest_string4 has the same behavior as longest_string2.
   - longest_string_helper has type (int * int -> bool) -> string list -> string (notice the currying). This function will look a lot like longest_string1 and longest_string2 but is more general because it takes a function as an argument.
   - longest_string3 and longest_string4 are defined with val-bindings and partial applications of longest_string_helper.

5. Write a function longest_capitalized that takes a string list and returns the longest string in the list that begins with an uppercase letter (or "" if there are no such strings). Use a val-binding and the ML library’s o operator for composing functions. Resolve ties like in problem 2.

6. Write a function rev_string that takes a string and returns the string that is the same characters in reverse order. Use ML’s o operator, the library function rev for reversing lists, and two library functions in the String module. (Browse the module documentation to find the most useful functions.)

The next two problems involve writing functions over lists that will be useful in later problems.

7. Write a function first_answer of type ('a -> 'b option) -> 'a list -> 'b (notice the 2 arguments are curried). The first argument should be applied to elements of the second argument until the first time it returns SOME v for some v and then v is the result of the call to first_answer. If the first argument returns NONE for all list elements, then first_answer should raise the exception NoAnswer. Hints: Sample solution is 5 lines and does nothing fancy.

8. Write a function all_answers of type ('a -> 'b list option) -> 'a list -> 'b list option (notice the 2 arguments are curried). The first argument should be applied to elements of the second argument. If it returns NONE for any element, then the result for all_answers is NONE. Else the calls to the first argument will have produced SOME lst1, SOME lst2, ..., SOME lstn and the result of all_answers is SOME lst where lst is lst1, lst2, ..., lstn appended together (order doesn’t matter). Hints: The sample solution is 8 lines. It uses a helper function with an accumulator and uses @. Note all_answers f [] should evaluate to SOME [].
The remaining problems use these type definitions, which are similar to ML-style pattern matching:

```plaintext
datatype pattern = Wildcard | Variable of string | UnitP | ConstP of int 
  | TupleP of pattern list | ConstructorP of string * pattern

datatype valu = Const of int | Unit | Tuple of valu list | Constructor of string * valu
```

Given `valu v` and `pattern p`, either `p matches v` or not. If it does, the match produces a list of `string * valu` pairs; order in the list does not matter. The rules for matching should be unsurprising:

- **Wildcard** matches everything and produces the empty list.
- **Variable s** matches any value `v` and produces the one-element list holding `(s,v)`.
- **UnitP** matches only `Unit`.
- **ConstP 17** matches only `Const 17` (and similarly for other integers).
- **TupleP ps** matches a value of the form `Tuple vs` if `ps` and `vs` have the same length and for all `i`, the `i`th element of `ps` matches the `i`th element of `vs`. The list produced is all the lists from the nested pattern matches appended together.
- **ConstructorP(s1,p)** matches `Constructor(s2,v)` if `s1` and `s2` are the same string (you can compare them with `=`) and `p` matches `v`. The list produced is the list from the nested pattern match.
- Nothing else matches.

9. (This problem uses the `pattern` datatype but is not really about pattern-matching.)

   (a) A function `g` has been provided to you. In an ML comment, describe in a few English sentences the arguments that `g` takes and what `g` computes (not how `g` computes it, though you will have to understand that to determine what `g` computes). Note you write no code for this subproblem.

   (b) Use `g` to define a function `count_wildcards` that takes a pattern and returns how many `Wildcard` patterns it contains.

   (c) Use `g` to define a function `count_wild_and_variable_lengths` that takes a pattern and returns the sum of the number of `Wildcard` patterns it contains and the string lengths of all the variables in the variable patterns it contains. (Use `String.size`.)

   (d) Use `g` to define a function `count_some_var` that takes a string and a pattern (as a pair) and returns the number of times the string appears as a variable in the pattern.

10. Write a function `check_pat` that takes a pattern and returns true if and only if all the variables appearing in the pattern are distinct from each other (i.e., use different strings). Note the choice of strings for constructors does not matter. Hints: The sample solution uses two helper functions. The first takes a pattern and returns a list of all the strings it uses for variables. Using `foldl` with a function that uses append is useful in one case. The second takes a list of strings and decides if it has repeats. `List.exists` may be useful. Sample solution is 15 lines.

11. Write a function `match` that takes a `valu * pattern` and returns a `(string * valu) list option`, namely `NONE` if the pattern does not match and `SOME lst` where `lst` is the list of bindings if it does. Hints: Sample solution has one case expression with 7 branches. The branch for tuples uses `all_answers` and `ListPair.zip`. Sample solution is 13 lines.

12. Write a function `first_match` that takes a value and a list of patterns and returns a `(string * valu) list option`, namely `NONE` if no pattern in the list matches or `SOME lst` where `lst` is the list of bindings for the first pattern in the list that matches. Hints: Sample solution is 3 lines and uses `first_answer` and a handle-expression.
Write a function `typecheck_patterns` that “type-checks” a pattern list. Types for our made-up pattern language are defined by:

```plaintext
datatype typ = Anything (* any type of value is okay *)
  | UnitT (* type for Unit *)
  | IntT (* type for integers *)
  | TupleT of typ list (* tuple types *)
  | Datatype of string (* some named datatype *)
```

`typecheck_patterns` should have type `((string * string * typ) list) * (pattern list) -> typ option`. The first argument contains elements that look like (“foo”,“bar”,IntT), which means constructor foo makes a value of type Datatype “bar” given a value of type IntT. Assume list elements all have different first fields (the constructor name), but there are probably elements with the same second field (the datatype name). Under the assumptions this list provides, you “type-check” the pattern list to see if there exists some typ (call it t) that all the patterns in the list can have. If so, return SOME t, else return NONE.

You must return the “most lenient” type that all the patterns can have. For example, given patterns `TupleP[Variable("x"),Variable("y")]
and TupleP[Wildcard,Wildcard]`, return `TupleT[Anything,Anything]` even though they could both have type `TupleT[IntT,IntT]`. As another example, if the only patterns are `TupleP[Wildcard,Wildcard]` and `TupleP[Wildcard,TupleP[Wildcard,Wildcard]]`, you must return `TupleT[Anything,TupleT[Anything,Anything]]`

Warning: The sample solution does not include the challenge problem.

**Type Summary:** Evaluating a correct homework solution should generate these bindings, in addition to the bindings for type and exception definitions:

```plaintext
val only_capitals = fn : string list -> string list
val longest_string1 = fn : string list -> string
val longest_string2 = fn : string list -> string
val longest_string3_helper = fn : (int * int -> bool) -> string list -> string
val longest_string3 = fn : string list -> string
val longest_string4 = fn : string list -> string
val longestcapitalized = fn : string list -> string
val rev_string = fn : string -> string
val g = fn : (unit -> int) -> (string -> int) -> pattern -> int
val count_wildcards = fn : pattern -> int
val count_wild_and_variable_lengths = fn : pattern -> int
val count_some_var = fn : string * pattern -> int
val first_answer = fn : ('a -> 'b option) -> 'a list -> 'b
val all_answers = fn : ('a -> 'b list option) -> 'a list -> 'b list option
val check_pat = fn : pattern -> bool
val match = fn : valu * pattern -> (string * valu) list option
val first_match = fn : valu -> pattern list -> (string * valu) list option
```

Of course, generating these bindings does not guarantee that your solutions are correct. Test your functions: Put your testing code in a second file. We will not grade it, but you must turn it in.

**Assessment:** Your solutions should be correct, in good style, and use only features we have used in class.

**Turn-in Instructions**

- Put all your solutions in one file, `lastname_hw3.sml`, where `lastname` is replaced with your last name.
  Put tests you wrote in `lastname_hw3_test.sml`.
- Turn in your file using the Catalyst dropbox link on the course website.