We continue describing programming idioms enabled by function closures.

Implementing an Abstract Data Type

The key to an abstract data type is requiring clients to use it via a collection of functions rather than directly accessing its private state. Thanks to this abstraction, we can later change how the data type is implemented without changing how it behaves for clients. In an object-oriented language, you might implement an abstract data type be defining a class with all private fields (inaccessible to clients) and some public methods (the interface with clients). We can do the same thing in ML with a record of closures; the variables that the closures use from the environment correspond to the private fields. Admittedly, this programming idiom can appear very fancy/clever/subtle, but it suggests (correctly) that functional programming and object-oriented programming are more similar than they might first appear (a topic we will revisit later in the course; there are important differences).

As an example, consider an implementation of a set of integers that supports creating a new bigger set and seeing if an integer is in a set. Our sets are mutation-free in the sense that adding an integer to a set produces a new, different set. In ML, we could define a type that describes this interface:

```plaintext
datatype set = S of { add : int -> set, member : int -> bool }
```

Roughly speaking, a set is a record with two fields, each of which holds a function. It would be simpler to write:

```plaintext
type set = { add : int -> set, member : int -> bool }
```

but this does not work in ML because type bindings cannot be recursive. So we have to deal with the mild inconvenience of having a constructor S around our record of functions defining a set. Notice we are not using any new types or features; we simply have a type describing a record with fields named add and member, each of which holds a function.

Once we have an empty set, we can use its add field to create a one-element set, and then use that set’s add field to create a two-element set and so on. So the only other thing our interface needs is a binding like this:

```plaintext
val empty_set = ... : set
```

Before implementing this interface, let’s see how a client might use it:

```plaintext
fun use_a_set () =
  let val S s1 = empty_set
      val S s2 = (#add s1) 34
      val S s3 = (#add s2) 19
  in
    if (#member s3) 42
    then 99
    else if (#member s3) 19
          then 17
          else 0
  end
```

Again we are using no new features. #add s1 is reading a record field, which in this case produces a function that we can then call with 34. If we were in Java, we might write s1.add(34) to do something similar. The val bindings use pattern-matching to “get rid of” the S constructors on values of type set.

There are many ways we could define empty_set; they will all use the technique of using a closure to “remember” what elements a set has. Here is one way:
val empty_set = 
  let fun exists(j,lst) = (* could use currying and/or fold to be fancier *)
      case lst of
        [] => false
      | hd::tl => j=hd orelse exists(j,tl)
    in
    make_set []
  end

The helper function \texttt{exists} just sees if a list has an element; we could also just use \texttt{List.exists} in the standard library. All the fanciness is in \texttt{make_set} and \texttt{empty_set} is just the record returned by \texttt{make_set \{\}}. What \texttt{make_set} returns is a value of type \texttt{set}. It is essentially a record with two functions. The closures produced from \texttt{fn i => make_set (i::lst)} and \texttt{fn i => exists (i,lst)} are values that \texttt{when called} use \texttt{lst} — which is the “private field” we need to produce either a \texttt{bool} (for \texttt{member}) or a new set (for \texttt{add}).

\textbf{Callbacks}

Another common idiom is to implement a library that detects when “events” occur and informs clients that have previously “registered” their interest in hearing about events. Clients can register their interest by providing a “callback” — a function that gets called when the event occurs. Examples of events for which you might want this sort of library include things like users moving the mouse or pressing a key. Data arriving from a network interface is another example.

The purpose of these libraries is to allow multiple clients to register callbacks. The library implementer has no idea what clients need to compute when an event occurs, and the clients may need “extra data” to do the computation. So the library implementer should not restrict what “extra data” each client uses. A closure is ideal for this because a function’s type \texttt{t1 \rightarrow t2} doesn’t specify the types of any other variables a closure uses, so we can put the “extra data” in the closure’s environment.

If you have used “event listeners” in Java’s Swing library, then you have used this idiom in an object-oriented setting. In Java, you get “extra data” by defining a subclass with additional fields. This can take an awful lot of keystrokes for a simple listener, which is a (the?) main reason the Java language added anonymous inner classes (which you do not need to know about for this course), which are closer to the convenience of closures.

To see an example in ML, we will finally introduce ML’s support for mutation. Mutation is okay in some settings. In this case, we really do want registering a callback to “change the state of the world” — when an event occurs, there are now more callbacks to invoke. In ML, most things really cannot be mutated. Instead you must create a \texttt{reference}, which is a container whose contents can be changed. You create a new reference with the expression \texttt{ref e} (the initial contents are the result of evaluating \texttt{e}). You get a reference \texttt{r}’s current contents with \texttt{!r} (not to be confused with negation in Java or C), and you change \texttt{r}’s contents with \texttt{r := e}. The type of a reference that contains values of type \texttt{t} is written \texttt{t ref}.

Our example will use the idea that callbacks should be called when a key on the keyboard is pressed. We will pass the callbacks an \texttt{int} that encodes which key it was. Our interface just needs a way to register callbacks. (In a real library, you might also want a way to unregister them.)

\texttt{val onKeyEvent : (int -> unit) -> unit}

Clients will pass a \texttt{int \rightarrow unit} that, when called later with an \texttt{int} will do whatever they want. To implement this function, we just use a reference that holds a list of the callbacks. Then when an event actually occurs, we assume the function \texttt{on_event} is called and it calls each callback in the list:

\texttt{val cbs : (int -> unit) list ref = ref []
fun onKeyEvent f = cbs := f::(!cbs)
fun on_event i =}
let fun loop l = 
case l of 
  [] => []
  | f::tl => f i; loop tl
in loop (!cbs) end

Most importantly, the type of \texttt{onKeyEvent} places no restriction on what extra data a callback can access when it is called. Here are three different clients (calls to \texttt{onKeyEvent} that use different variables in functions in their environment:

\begin{verbatim}
onKeyEvent (fn i => write_to_log(Int.toString i 
  ^ " got pressed
"));
\end{verbatim}

val f4_key = 75; (* no idea what it really is *)
onKeyEvent (fn i => if i=f4_key then minimizeWindow() else ());

fun prohibit_keys lst = 
onKeyEvent (fn i => if (List.exists (fn j => j=i) lst) 
  then exitProgram()
  else ());
prohibit_keys [13, 42, 99];

We describe the \texttt{List.exists} library function below.

**Partial application ("currying")**

The final idiom we consider is very convenient, especially when defining and using iterators over data structures (see last lecture). We have already seen that in ML every function takes exactly one argument, so you have to use an idiom to get the effect of multiple arguments. Previously we have done this by passing a tuple as the one argument, so each part of the tuple is conceptually one of the multiple arguments. Another more clever and often more convenient way is to have a function take the first conceptual argument and return another function that takes the second conceptual argument and so on.

This technique is called “currying” because someone named Curry was a researcher who studied the idea. Here is an example of a “3-argument” function that uses currying:

\begin{verbatim}
val inorder3 = fn x => fn y => fn z =>
  z >= y andalso y >= x
\end{verbatim}

If we call \texttt{inorder3 4} we will get a closure that has \texttt{x} in its environment. If we then call this closure with 5, we get a closures that has \texttt{x} and \texttt{y} in its environment. If we then call this closure with 6, we will get \texttt{true} because 6 is greater than 5 and 5 is greater than 4. That is just how closures work.

So \texttt{((inorder3 4) 5) 6} computes exactly what we want and feels pretty close to calling \texttt{inorder3} with 3 arguments. Even better, the parentheses are optional, so we can write exactly the same thing as \texttt{inorder3 4 5 6}, which is actually fewer non-space characters than our old tuple approach where we would have:

\begin{verbatim}
fun inorder3 (x,y,z) = z >= y andalso y >= x
val someClient = inorder3(4,5,6)
\end{verbatim}

Moreover, even though we might expect most clients of our curried \texttt{inorder3} to provide all 3 conceptual arguments, they might provide fewer and use the resulting closure later. This is called “partial application” because we are providing a subset (more precisely, a prefix) of the conceptual argument. As a silly example, \texttt{inorder3 0 0} returns a function that returns \texttt{true} if its argument is nonnegative.

Currying is particularly convenient for creating similar functions with iterators. For example, here is a curried version of a fold function for lists:
fun fold f = fn acc => fn l =>
  case l of
      [] => acc
    | hd::tl => fold f (f(acc,hd)) tl

Now we could use this fold to define a function that sum's a list elements like this:

fun sum1 l = fold ((fn (x,y) => x+y) 0 l

But that is unnecessarily complicated compared to just using partial application:

val sum2 = fold (fn (x,y) => x+y) 0

The convenience of partial application is why many iterators in ML's standard library use currying with the function they take as the first argument.

There is syntactic sugar for defining curried functions; you can just separate the conceptual arguments by spaces rather than using anonymous functions. So the better style for our fold function would be:

fun fold f acc l =
  case l of
      [] => acc
    | hd::tl => fold f (f(acc,hd)) tl

In our callback example, we used a library function List.exists, which uses currying. That is why its two conceptual arguments were separated by spaces. While it is good to use library functions where convenient, exists is quite easy to implement ourselves:

fun exists predicate lst =
  case lst of
      [] => false
    | hd::tl => predicate hd orelse exists predicate tl

Sometimes functions are curried but the arguments are not in the order you want for a partial application. Or sometimes a function is curried when you want it to use tuples or vice-versa. Fortunately our earlier idiom of combining functions can take functions using one approach and produce functions using another:

fun other_curry1 f = fn x => fn y => f y x
fun other_curry2 f x y = f y x
fun curry f x y = f (x,y)
fun uncurry f (x,y) = f x y

Looking at the types of these functions can help you understand what they do. As an aside, the types are also fascinating because if you pronounce -> as “implies” and * as “and”, the types of all these functions are logical tautologies.

Finally, you might wonder which is faster, currying or tupling. It almost never matters; they both do work proportional to the number of conceptual arguments, which is typically quite small. For the one or two performance-critical functions in your software, it might matter to pick the faster way. In the version of the ML compiler we are using, tupling happens to be faster. In other implementations of ML, curried functions are faster.