

Where are we

- Today: Notions of equivalence; probably start modules
- Monday: Modules; possibly start Scheme
- Wednesday: Scheme basics
- Thursday: Scheme pragmatics; some review
- Friday: midterm
 - Includes modules, but not Scheme
 - You can have one side of one 8.5x11 sheet of paper
 - Old midterms, etc. posted by Monday
 - Will read code, write code, and write English
 - Heavily biased toward later lectures because we have been building
 - (My exams are difficult; don't panic.)

Equivalence

"Equivalence" is a fundamental programming concept

- Code maintenance (simplify code)
- Backward-compatibility (add new optional features)
- Program verification (compare to reference version)
- Program optimization (make faster without breaking it)
- Abstraction and strong interfaces (coming soon!)

But what does it mean for an expression (or program) e1 to be "equivalent" to expression e2?

First equivalence notion

Context (i.e., "where equivalent")

- Given where e1 occurs in a program e, replacing e1 with e2 does not change e in any way
- At any point in any program, replacing e1 with e2 makes an equivalent program

The latter (contextual equivalence) is much more interesting.

For the former, the body of an unused function body is equivalent to everything (that typechecks).

Second equivalence notion

"how equivalent"

- "partial": e1 is equivalent to e2 if for any input, any output e1 produces is what e2 produces
- "total": partial plus $e\mathbf{1}$ must terminate if and only if $e\mathbf{2}$ terminates

Notice even total contextual equivalence ignores *efficiency* (an exponential algorithm could be "equivalent" to a linear algorithm) according to *this definition*.

Key notion: what is observable?

(For example: Is time-elapsed observable?)

Accounting for "Effects"

Consider whether fn x => e1 and fn x => e2 are totally contextually equivalent.

Is this enough? For any environment, e1 terminates and evaluates to v under the environment if and only if e2 terminates and evaluates to v under the environment.

We must also consider any *effects* the function may have.

• Mutation, exceptions, printing, modifying files, ...

Functional languages discourage function bodies that do exactly the things that destroy total contextual equivalence.

- For example, *if* you "stay functional" then (f x) + (f x) can be replaced by (f x)*2 *without* consulting what f is bound to.
- (Side)-effects are often worth discouraging in any language.

Function equivalences

There are 3 very general things you can do with functions that produce equivalent code. Recognizing them (and their subtle caveats) can make you a better programmer.

- 1. Systematic renaming of variables
- 2. "Inlining" by replacing a function call with a body + substitutions
- 3. Unnecessary function wrapping

Before considering each, it will help to define carefully the notion of *free variables*...

Free variables

An expression e has a set of *free variables*. The definition is:

- For each *use* of a variable, find the *binding* that defines that variable. (This uses the language's *scope rules*.)
- If there is a *use* of x that is in e whose *corresponding binding* is outside e, then x is in the free variables of e.

Example:

```
fun f x =

let val w = x + y

val y = fn x => z + y + x

val q = w + x

in if g w then x+4 else f (x-1) end
```



Scope matters

```
ls fn x => e1 is equivalent to fn y => e2 where e2 is e1 with every
x replaced by y?
What if e1 is y?
What if e1 is fn x => x?
Need caveats: fn x => e1 is equivalent to fn y => e2 where e2 is
e1 with every free x replaced by y. But only if y is not free in e1!
```



More scope mattering

Is (fn x => e1) e2 equivalent to e3 where e3 is e1 with every x
replaced by e2?

• Every *free* x (of course).

- Example: (fn x => (fn x => x)) 17

• A free variable in e2 must not be bound at an occurrence of x. (Called "capture".)

- Example: val y = 4; val z = (fn x => (fn y => x)) y

- Evaluating e2 must terminate, not do assignments, not raise exceptions, not print, etc.
 - Because in ML (but not all functional languages), e2 is evaluated *before* the call

- Example: (fn x => x+x) ((print "hi";5))

• Efficiency? Could be faster or slower. (Why?)

Unnecessary Function Wrapping

A common source of bad style for beginners

```
Is e1 equivalent to fn x \Rightarrow e1 x?
```

Sure, provided:

- e1 effect-free (terminates, no mutation, printing, exceptions, etc.)
- x does not occur free in e1

Example:

```
List.map (fn x => SOME x) lst
List.map SOME lst
```

Notice variables, constructors, etc. are bound to values, so they are always effect-free (the value is already computed)

Summary so far

We breezed through some core programming-language facts:

- Definition of equivalence depends on observable behavior
- Notion of free variables crucial to understanding function equivalence.
- Three forms of function equivalence:
 - Systematic Renaming
 - Inlining
 - Unnecessary Function Wrapping

Another notion of equivalence we have mentioned but not focused on: *syntactic sugar*

Syntactic Sugar

When all expressions using one construct are totally equivalent to another more primitive construct, we say the former is "syntactic sugar".

- Makes language definition easier
- Makes language implementation easier

Examples:

- e1 andalso e2 (define as a conditional)
- if e1 then e2 else e3 (define as a case)
- tuples are really records with field names 1, 2, ...

Note: The error messages used to be even worse because the type-checker worked on a desugared version of your code.

