Function-Call Efficiency

First: Function calls take constant \( O(1) \) time, so until you’re using the right algorithms and have a critical bottleneck, forget about it.

That said, ML’s “all functions take one argument” can be inefficient in general:

- Create a new \( n \)-tuple
- Create a new function closure

In practice, implementations optimize common cases. In some implementations, \( n \)-tuples are faster (avoid building the tuple). In others, currying is faster (avoid building intermediate closures).

In the \(< 1\) percent of code where detailed efficiency matters, you program against an implementation. Worrying about this stuff at the wrong stage and for the wrong code is a bad idea.

Equivalence

“Equivalence” is a fundamental programming concept

- Code maintenance / backward-compatibility
- Program verification
- Program optimization
- Abstraction and strong interfaces

But what does it mean for an expression (or program) \( e_1 \) to be “equivalent” to expression \( e_2 \)?
Equivalence I: where?

Context (i.e., "where equivalent")

- Given where \( e_1 \) occurs in a program \( e \), replacing \( e_1 \) with \( e_2 \) makes a program \( e' \) equivalent to \( e \)
- At any point in any program, replacing \( e_1 \) with \( e_2 \) makes an equivalent program.

The latter (contextual equivalence) is much more interesting.

For the former, the body of an unused function body is equivalent to everything (that typechecks).

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Equivalence II: how?

"how equivalent"

- "partial": \( e \) and \( e' \) are partially equivalent if on any input where both give an output, they give the same output
- "total": partial plus \( e \) and \( e' \) have the same termination behavior
- efficiency: \( e \) and \( e' \) are totally equivalent and one never takes more than (for example) \( c \) times longer than the other (or uses much more space or ...)
- syntactic notions: \( e \) and \( e' \) differ only in whitespace and comments (for example)

Key notion: what is observable? (memory, clock, REP-loop, file-system, ...)

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Accounting for "Effects"

Consider whether \( \text{fn} \ x \implies a_1 \) and \( \text{fn} \ x \implies a_2 \) are totally contextually equivalent.

Is this enough? For all environments, \( e_1 \) terminates and evaluates to \( v \) under the environment if and only if \( e_2 \) terminates and evaluates to \( v \) under the environment.

Functions produce values; may also produce (side-) effects. Consider both!

Purely functional languages have fewer/none, but ML is not purely functional (mutation, exceptions, printing, files, ...)

In real languages, contextual equivalence usually requires many things.

Nonetheless, "equivalence" usually means total contextual equivalence for practical purposes (optimization, reasoning about correctness, etc.).

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Syntactic Sugar

When all expressions using one construct are totally equivalent to another more primitive construct, we say the former is "syntactic sugar".

- Makes language definition, use & implementation easier (for varying reasons)
- (But may make error messages more cryptic...)

Examples:

\[
\begin{align*}
e_1 \& \text{andalso} \quad & e_2 & \quad \mapsto & \text{if } e_1 \text{ then } e_2 \text{ else } \text{false} \\
\text{if } e_1 \text{ then } e_2 \text{ else } e_3 & \quad \mapsto & \text{case } e_1 \text{ of } \text{true} & \Rightarrow e_2 \mid \text{false} & \Rightarrow e_3 \\
\text{fun } f \ x \ y = e & \quad \mapsto & \text{val } f = \text{fn} \ x \ y & \Rightarrow e \\
\text{#_ } e & \quad \mapsto & \text{let val } (x, \ldots) & \Rightarrow e \text{ in } x \text{ end}
\end{align*}
\]
More sugar

If we ignore types, then we have even more sugar:

```haskell
let val p = e1 in e2 end -> (fn p => e2) e1
```

In fact, if we let every program type-check (or just use one big datatype), then a language with just functions and function application is theoretically as powerful as ML or Java.

Even without data!

( [] ≠ [[]] ≠ [[[]]] ≠ [ [], [[]], [] ] ... suffice)

This language is called "lambda calculus" – we'll learn a bit more about it later.

Equivalences for Functions

While sugar defines one construct in terms of another, there are also important notions of meaning-preserving changes involving functions and bound variables.

They're so important that a goal of language design is that a language supports them.

But the correct definitions are subtle.

First example: systematic renaming

Is `fn x => e1` equivalent to `fn y => e2` where `e2` is `e1` with every `x` replaced by `y`?

Systematic renaming requires care

Is `fn x => e1` is equivalent to `fn y => e2` where `e2` is `e1` with every `x` replaced by `y`?

What if `e1` is `y`? (More generally, contains free `y`?)

What if `e1` is `fn x => z`? (More generally, contains bound `x`?)

Need caveats: `fn x => e1` is equivalent to `fn y => e2` where `e2` is `e1` with every free `x` replaced by `y` and `y` is not free in `e1`, and no free `x` occurs within the scope of a binding for `y` (capture; e.g.:

```haskell
fn x => let y = 2 in x - y end; see also next slide.
```

Note: We can provide a very precise recursive (meta-)definition of free variables in an expression.

Next: Is `fn x => e1` `e2` equivalent to `e3` where `e3` is `e1` with every `x` replaced by `e2`?

- Every free `x` (of course).
- A free variable in `e2` must not be bound at an occurrence of `x`.
  (Called "capture".)
  - Always satisfiable by renaming bound variables.
- Evaluating `e2` must have no effects (printing, exceptions, infinite-loop, etc.)
  - Closely tied to the rule that arguments are evaluated to values before function application. (Not true for all languages)
  - In ML, many expressions have no such effects (`x`, `foo x`, ...); much fewer in Java.
- Efficiency? Could be faster or slower; more or less space. (Why?)

Argument Substitution ("Inlining")

Is `(fn x => e1) e2` equivalent to `e3` where `e3` is `e1` with every `x` replaced by `e2`?

- Every free `x` (of course).
- A free variable in `e2` must not be bound at an occurrence of `x`.
  (Called "capture".)
  - Always satisfiable by renaming bound variables.
- Evaluating `e2` must have no effects (printing, exceptions, infinite-loop, etc.)
  - Closely tied to the rule that arguments are evaluated to values before function application. (Not true for all languages)
  - In ML, many expressions have no such effects (`x`, `foo x`, ...); much fewer in Java.
- Efficiency? Could be faster or slower; more or less space. (Why?)
Unnecessary Function Wrapping

A common source of bad style for beginners

Is e₁ equivalent to \( \text{fn } x \Rightarrow e₁ x \)?

Sure, provided:

- e₁ is effect-free
- x does not occur free in e₁

Example:

\[
\begin{align*}
\text{map (fn x => SOME x) 1st} \\
\text{map SOME 1st}
\end{align*}
\]

Summary

- Definition of equivalence depends on observable behavior
- Syntactic sugar “makes a big language smaller” by defining constructs in terms of equivalence
- Notion of free and bound variables crucial to understanding function equivalence.
- Three common forms of function equivalence:
  - Systematic Renaming
  - Argument Substitution
  - Unnecessary Function Wrapping