So far...

map : given F and list $[a_1, a_2 \ldots a_n]$, produces $[F(a_1), F(a_2), \ldots F(a_3)]$

reduce : given F and list $[a_1, a_2 \ldots a_n]$, produces $F(a_1, F(a_2, F(\ldots, F(a_{n-1}, a_n) \ldots)))$

filter : given predicate P and list $[a_1, a_2 \ldots a_n]$, produces elements in the given list that satisfy predicate P.

```
fun map (F,nil) = nil
  | map (F,x::xs) = F(x)::map(F,xs)

exception EmptyList;
fun reduce(F, nil) = raise EmptyList
  | reduce(F, [a]) = a
  | reduce(F, x::xs) = F(x, reduce(F, xs));

fun filter(P, nil) = nil
  | filter(P, x::xs) =
      if P(x) then x::filter(P, xs)
      else filter(P,xs)
```

Question: what is the type of these functions?

Mini-exercise:

$$\prod_{i=1}^{5} \frac{x}{i} \tag{1}$$

hint: use map and reduce

A relaxing exercise to wake you up...

Question: Write a function `tabulate` that as arguments an initial value a, an increment $delta$, a number of points n, and a function F of type (real→real).

Return a list of two-tuples (x, $F(x)$) where x $= a, a + delta, a + 2 * delta, \ldots, a + (n - 1) * delta$

Side Note 1: Try not using parentheses on your function arguments. ex: *fun F x* instead of *fun F(x)*.

Side Note 2: What will the type be?

```
fun tabulate a delta n F =
let
    fun t i result =
    let
        val x = a+real(i)*delta
    in
        if i=n
        then result
        else t (i+1) (result@[(x, F(x))])
    end
in
    t 0 []
end
```

Calculus

No, you're not in the wrong classroom

Derivative

The derivative of a function f with respect to x is denoted $f'(x)$ or $\frac{df}{dx}$, which is defined as

$$f'(x)\frac{f(x+h)-f(x)}{h}$$

or more symmetrically as

$$f'(x)\frac{f(x+h)-f(x-h)}{2h}$$

To do numerical differentiation, simply pick some very small h, say, $1E-6$.

Some nostalgic examples:

$\frac{d}{dx}x^n$

$\frac{d}{dx}\ln x$

$\frac{d}{dx}\sin x$

$\frac{d}{dx}e^x$

Integral

Definite integral: An integral $\int_a^b f(x)dx$ with upper and lower limits.

Indefinite integral (antiderivative): An integral of the form $\int f(z)dz$, that is, without upper and lower limits.

There are $tons$ of ways to do numerical integration (and you probably know better than I do), but we'll stick to the simplest one (and the only way I can understand without re-reading Calculus), which is the **trapezoidal rule**

$$\int_a^b f(x)dx \approx (b-a)\frac{f(a)+f(b)}{2}$$

For a more accurate approximation, we can break up the interval $[a, b]$ into n subintervals

$$\int_a^b f(x)dx \approx \frac{b-a}{n}(\frac{f(a)+f(b)}{2} + \sum_{k=1}^{n-1} f(a + k\frac{b-a}{n}))$$