So far...

*map*: given $F$ and list $[a_1, a_2 \ldots a_n]$, produces $[F(a_1), F(a_2), \ldots F(a_n)]$

*reduce*: given $F$ and list $[a_1, a_2 \ldots a_n]$, produces $F(a_1, F(a_2, F(\ldots, F(a_{n-1}, a_n)\ldots)))$

*filter*: given predicate $P$ and list $[a_1, a_2 \ldots a_n]$, produces elements in the given list that satisfy predicate $P$. 
fun map (F,nil) = nil
  | map (F,x::xs) = F(x)::map(F,xs)

exception EmptyList;
fun reduce(F, nil) = raise EmptyList
  | reduce(F, [a]) = a
  | reduce(F, x::xs) = F(x, reduce(F, xs));

fun filter(P, nil) = nil
  | filter(P, x::xs) =
      if P(x) then x::filter(P, xs)
      else filter(P,xs)

Question: what is the type of these functions?

Mini-exercise:
\[
\prod_{i=1}^{5} \frac{x}{i} \tag{1}
\]
hint: use map and reduce
A relaxing exercise to wake you up...

Question: Write a function \texttt{tabulate} that as arguments an initial value \( a \), an increment \( \text{delta} \), a number of points \( n \), and a function \( F \) of type (real\( \rightarrow \)real).

Return a list of two-tuples \((x, F(x))\) where \( x = a, a + \text{delta}, a + 2 \times \text{delta}, \ldots, a + (n - 1) \times \text{delta} \)

\textbf{Side Note 1:} Try not using parentheses on your function arguments. ex: \texttt{fun F x} instead of \texttt{fun F(x)}.

\textbf{Side Note 2:} What will the type be?
fun tabulate a delta n F = 
  let
    fun t i result = 
      let
        val x = a+real(i)*delta
      in
        if i=n
        then result
        else t (i+1) (result@[(x, F(x))])
      end
    in
    t 0 []
  end
Calculus

No, you’re not in the wrong classroom
Derivative

The derivative of a function $f$ with respect to $x$ is denoted $f'(x)$ or $\frac{df}{dx}$, which is defined as

$$f'(x) = \frac{f(x+h)-f(x)}{h}$$

or more symmetrically as

$$f'(x) = \frac{f(x+h)-f(x-h)}{2h}$$

To do numerical differentiation, simply pick some very small $h$, say, $1E-6$.

Some nostalgic examples:

$$\frac{d}{dx} x^n$$

$$\frac{d}{dx} \ln x$$

$$\frac{d}{dx} \sin x$$

$$\frac{d}{dx} e^x$$
Integral

**Definite integral:** An integral \( \int_{a}^{b} f(x) \, dx \) with upper and lower limits.

**Indefinite integral (antiderivative):** An integral of the form \( \int f(z) \, dz \), that is, without upper and lower limits.

There are *tons* of ways to do numerical integration (and you probably know better than I do), but we’ll stick to the simplest one (and the only way I can understand without re-reading Calculus), which is the **trapezoidal rule**

\[
\int_{a}^{b} f(x) \, dx \approx (b - a) \frac{f(a) + f(b)}{2}
\]

For a more accurate approximation, we can break up the interval \([a, b]\) into \(n\) subintervals

\[
\int_{a}^{b} f(x) \, dx \approx \frac{b-a}{n} \left( \frac{f(a) + f(b)}{2} + \sum_{k=1}^{n-1} f(a + k \frac{b-a}{n}) \right)
\]