Today

• We have learned an interesting subset of the ML expression language

• But we have been really informal about some aspects of the type system:
  – Type inference (what types do bindings implicitly have)
  – Type variables (what do ’a and ’b really mean)
  – Type constructors (why is int list a type but not list)

• Note: Type inference and parametric polymorphism are separate concepts that end up intertwined in ML. A different language could have one or the other.
Type Inference

Some languages are untyped or dynamically typed.

ML is *statically typed*; every binding has one type, determined during type-checking (compile-time).

ML is *implicitly typed*; programmers rarely need to write the types of bindings.

The type-inference question: Given a program without explicit types, produce types for all bindings such that the program type-checks, or reject (only) if it is impossible.

Whether type inference is easy, hard, or impossible depends on details of the type system: Making it more or less powerful (i.e., more programs typecheck) may make inference easier or harder.
ML Type Inference

- Determine types of bindings in order (earlier first) (except for mutual recursion)

- For each `val` or `fun` binding, analyze the binding to determine necessary facts about its type.

- Afterward, use type variables (e.g., `'a`) for any unconstrained types in function arguments or results.

- Some extra details for type variables and references we’ll mention later.

Amazing fact: For the ML type system, “going in order” this way never causes unnecessary rejection.
Example 1

fun f x =
    let val (y,z) = x in
        (Real.abs y) + z
    end
Example 2

fun sum lst =
    case lst of
        [] => 0
    | hd::tl => hd + (sum tl)
Example 3

fun compose (f, g, x) = f (g x)
Comments on ML type inference

• If we had subtyping, the “equality constraints” we generated would be unnecessarily restrictive.

• If we did not have type variables, we would not be able to give a type to compose until we saw how it was used.
  – But type variables are useful regardless of inference.

• Inference is why the following aren’t really equivalent:
  – let val x = e1 in e2 end
  – (fn x => e2) e1

E.g., let’s try e2 = (x 0, x "foo") and something simple for e1 like fn y => y:
  – let val x = (fn y => y) in (x 0, x "foo") end
  – (fn x => (x 0, x "foo")) (fn y => y)

The latter gives a type error ...
Parametric polymorphism

Fancy words for “forall-types”. Coming to next version of Java, C#, VB, etc. Sometimes called generics. A bit like C++ templates if C++ didn’t have operator-overloading.

In principle, just two new kinds of types:

\[ \text{tv} ::= 'a | 'b | ... \]
\[ \text{t} ::= \text{int} | \text{string} | \text{bool} | \text{t1->t2} | \{l1:t1, ..., ln:tn}\]
 \[ \text{dtname} | \text{tv} | \forall 'tv. \text{t} \]

Given an expression of type \( \forall 'tv. \text{t} \), we can instantiate it at type \( \text{t2} \) to get an expression of type “\( \text{t with 'tv replaced by t2} \)”

Example: We can instantiate

\( \forall 'a. \forall 'b. ('a * 'b) -> ('b * 'a) \)

with string for \( 'a \) and int->int for \( 'b \) to get

\( \text{string} * (\text{int->int}) \) -> ((\text{int->int}) * \text{string}) \)
ML-style polymorphism

The ML type system is actually more restrictive:

- “forall” must appear “all the way on the outside-left”
- So it’s implicit; no way to write the words “for all”

Example: (‘a * ‘b) -> (‘b * ‘a) means
forall ‘a. forall ‘b. (‘a * ‘b) -> (‘b * ‘a)

Non-example: There’s no way to have a type like
int -> (forall ‘a. ‘a -> int)
Versus Subtyping

Compare

fun swap (x,y) = (y,x) (* ('a * 'b) -> ('b * 'a) *)

with

class Pair { Object x; Object y; ... }
Pair swap(Pair pr) { return new Pair(pr.y, pr.x); }

ML wins in two ways (for this example):

- Caller instantiates types, so doesn’t need to cast result
- Callee cannot return a pair of any two objects.
Containers

Parametric polymorphism (for all types) are also the right thing for containers (lists, sets, hashtables, etc.) where elements have the same type.

Example: ML lists

```ml
val :: : ('a * ('a list)) -> 'a list (* infix is syntax *)
val map : (('a -> 'b) * ('a list)) -> 'b list
val sum : int list -> int
val fold : ('a * 'b -> 'b) -> 'b -> ('a list) -> 'b
```

List is a type constructor, not a type; if t is a type, then t list is a type.
User-defined type constructors

Language-design: don't provide a fixed set of a useful thing.
Let programmers declare type constructors.

Examples:

```
datatype 'a non_mt_list = One of 'a
  | More of 'a * ('a non_mt_list)

datatype 'a rope = Empty
  | Cons of 'a * ('a rope)
  | Rope of ('a rope) * ('a rope)
```

You can have multiple type-parameters (not shown here).

And now, finally, *everything* about lists is syntactic sugar!