Example 2

```haskell
val x = 1
fun f y = let val x = 2 in fn z => x + y + z end
val x = 3
val g = f 4
val y = 5
g 6
```

Example 3

```haskell
fun f g = let val x = 3 in g 2 end
val x = 4
fun h y = x + y
f h
```

Scope

A key language concept: how are user-defined things resolved?

We have seen that ML has lexically scoped variables

Another (more-antiquated-for-variables, sometimes-useful) approach is dynamic scope

Example of dynamic scope: Exception handlers (where does raise transfer control?)

Another example of dynamic scope: shell commands and shell scripts (environment variables)

The more restrictive "no free variables" makes important idioms impossible.
Why lexical scope?

1. Functions can be reasoned about (defined, type-checked, etc.) where defined
2. Function meaning not related to choice of variable names
3. “Closing over” local variables creates private data; function definer knows function users do not depend on it

Example:

```
fun add_2x x = fn z => z - x - x

fun add_2x x = let val y = x - x in fn z => z + y end
```

Key idioms with closures

- Create similar functions
- Pass functions with private data to iterators (map, fold, ...)
- Combine functions
- Provide an ADT
- As a callback without the “wrong side” specifying the environment.
- Partially apply functions (“currying”)

Create similar functions

```
val addn = fn n => fn m => n-m
val increment = addn 1
val add_two = addn 2
fun f n = 
  if n=0
    then []
  else (addn n)::(f (n-1))
```

Private data, for map/fold

Previously we saw map, this fold function is even more useful:

```
fun fold (f,acc,l) =
  case l of
    [] => acc
  | hd::tl => fold (f, f(acc,hd), tl)
```

Example uses (without using private data):

```
fun f1 l = fold ((fn (x,y) => x+y), 0, l)
fun f2 l = fold ((fn (x,y) => x andalso y >= 0), true, l)
```

Example use (with private data):

```
fun f3 (l,lo,hi) =
  fold ((fn (x,y) =>
    if y >= lo andalso y <= hi then x | l else x),
    0, l)
```
More on fold and private data

Another more general example:

\[
\text{fun } f_4 \ (l,g) = \text{fold } (\ (\text{fn } (12,y) \Rightarrow (g \ y); 12), \ [], \ 1)\]

A fold function over a data structure is much like a \textit{visitor pattern} in OOP.

We define fold once and do not restrict the type of the function passed to fold or the environment in which it is defined.

In general, libraries should not unnecessarily restrict clients.

Combine functions

\[
\text{fun } f_1 \ (g,h) = \text{fn } x \Rightarrow g \ (h \ x) \\
\text{fun } f_2 \ (g,h) = \text{fn } x \Rightarrow \\
\quad \text{case } g \ x \text{ of NONE } \Rightarrow h \ x | \text{ SOME } y \Rightarrow y
\]